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A *Proprium* (or property) of a genus, species, or individual is any attribute which follows from its comprehension either deductively or causally. If it follows from the comprehension of the genus, the property is called *generic;* if from that of the species, *specific;* and if from that of the individual object, *individual.* Thus, an individual thing may have its individual property, its specific property, or a property following from the species to which the individual belongs, and even a generic property following from the genus to which its species belongs. This last may be included in the specific property. A species may have two properties, one following from its differentia, and the other from its genus. The former is called the specific, and the latter the generic property, of the species; or both together are simply called its *property.* 'Memory,' for example, may be regarded as a property of man, following either from the genus animal, or from the differentia rational; 'power of judging' is likewise a property of man following from the differentia. The properties of the triangle, as proved in the *Elements* of Euclid, follow partly from the comprehension of its genus figure, partly from that of triangle, and partly from those of special kinds of triangles.

An *Accidens* (or accident) of an individual, genus, or species is any attribute which is possessed by it, and which does not follow from, or form a part of, its comprehension. If an accidens always belongs to an individual, or if it belongs to all the members of a genus, or species, it is called an *inseparable accidens* of that individual, genus, or species; as the place or date of birth of a particular person, the hair of man, the blackness of the crow, the whiteness of snow, &c. If, on the other hand, an accidens is sometimes present and sometimes absent in an individual, or if it belongs to a part only of a species or genus, then it is called a *separable accidens* of that individual, species, or genus; as the walking or sitting of a particular person, the wisdom of man, the solubility in water of salts, the opacity of gases, the learning of man, &c.

When the predicate of a proposition is a proprium, or an

accidens, of the subject, the latter in extension is included in the former, that is, the extension of the accidens or proprium, when taken as a general term, is a greater whole than that of the subject; while, in comprehension, the predicate expresses an attribute not contained in the connotation of the subject, that is, it imparts some new information about it; and the proposition, therefore, belongs to the class of real. In the proposition "Water boils at 100° C., under a pressure of 760 mm.," the attribute expressed by the predicate is not a part of the connotation of the term water.

The five terms—genus, species, differentia, proprium, and accidens—are called predicables, because whatever may be predicated (affirmed) of a subject in a proposition is, in relation to the subject, one or other of the five. A predicable is thus a name of a class of predicates in relation to the subjects. It should be distinguished, on the one hand, from the word 'predicament,' or 'category,' which means a most general class of both subjects and predicates, and, on the other, from the word 'predicate,' which means what is affirmed or denied of a subject. Given a term: whatever be affirmed of it, the predicate, *in relation* to the subject, is a predicable, that is, it is either a genus, species, differentia, proprium, or accidens; and the subject as well as the predicate must belong to some category or other. Aristotle gave four predicables, *viz.*, genus, definition, proprium, and accidens. Later logicians added 'species' and 'differentia' to Aristotle's list, and removed 'definition' from it. Thus there came to be the five predicables we have explained above. Some logicians have made further additions to the list. Professor Fowler, for example, gives 'synonym,' 'definition,' 'designation,' *idion* (a Greek word signifying a peculiar property), in addition to the five, while others regard them as falling under one or other of the five predicables adopted by them: 'synonym' and 'designation,' for example, would be regarded by some of them as included in accidens, 'definition' as a compound of genus and differentia, and 'idion' as coming under either differentia or property.

Besides the terms explained above, the older logicians use the term *summum genus* to mean a highest genus or a genus which cannot be a species, being the highest and most general of its kind, and the term *infima species* to mean a lowest species or a class which cannot be a genus to another, being the lowest of its kind, while the intermediate genera and species are called by them *subaltern genera* and *species.* 'Substance,' for example, is regarded by them as a summum genus, 'man' as an infima species, incapable of further subdivision into species, and 'body,' 'living being,' and 'animal' as subaltern genera and species.

The two terms 'genus' and 'species' express the relation of containing and contained. Any class containing another is popularly called a genus in relation to the latter, which is called a species. In the Sciences of Classification, in Botany and Zoology, for example, groups of a particular description are called genera in relation to others of an equally definite nature, which are called species. In order to express the relation of containing and contained, we not only use the two old terms, genus and species, but also many others according to the position of the groups in a system of division or classification. For example, the terms kingdom and sub-kingdom, class and sub-class, order and sub-order, genus and sub-genus, species and sub-species, variety and sub-variety, used in Zoology and Botany, mark as clearly the relation of containing and contained as the two words, genus and species.

Exercises.

I. State whether the following propositions are verbal or real, analytical or synthetical, and whether the predicate in relation to the subject is a genus, species, differentia, proprium, or accidens:—

1. Oxygen is an elementary gas.
2. Water boils at 100° C., under a pressure of 760 mm.
3. Platinum is a rare metal.
4. Sugar is sweet.
5. The atmospheric air is a mixture of nitrogen and oxygen.

6. Copper conducts heat as well as electricity.
7. All men have the power of thinking.
8. All animals are sentient beings.
9. All the flowering plants have fruits.
10. Heat expands bodies.
11. The leaves of plants are green.
12. Spring-water contains many salts in solution.
13. Hydrogen is the lightest substance known.
14. London is the largest city in England.
15. Milton was blind when he composed the "Paradise Lost."

II. Give the genus, species, differentia, proprium, and accidens of each of the following terms:—

(1) Triangle, (2) Circle, (3) Straight line, (4) Square, (5) Right angle, (6) Element, (7) Force, (8) Material Body, (9) Animal, (10) Chalk, (11) Rock, (12) Virtue, (13) Volition, (14) Knowledge, (15) Pleasure.

§ 11. Miscellaneous Exercises on Propositions.

In describing the logical characters of a proposition, the following method should be followed:—

I. What is given is a sentence. Ascertain whether the sentence consists of a single proposition or of a plurality of propositions.

II. In the former case, state whether it is—

i. Categorical, Hypothetical, or Disjunctive.
ii. Affirmative or Negative.
iii. Necessary, Assertory, or Problematic.
iv. Universal, Particular, or Indesignate; Singular and Universal, or Singular and Particular.
v. Verbal (or Analytical) or Real (or Synthetical).

Both the quality and quantity of a proposition may also be stated at once by saying whether it is A, E, I, or O.

III. In the latter case, state the propositions of which it consists, and treat each of them as detailed above.

IV. Sometimes the quality, quantity, and other characters of a proposition are not quite evident from its form or the manner of its statement. In such cases, verbal changes should be made in order to state it in the logical form, keeping the meaning the same. It is always safe first to ascertain, as in the case of the term, the meaning

of the proposition, or, where this is not practicable, to see, before attempting to describe the logical characters of the proposition, whether the subject be a general term taken distributively or not, whether there be any negative particle attached to the copula or to the predicate, whether there are any signs of universality or negation before the subject, &c.

Examples.

1. "No man is perfect": categorical, negative, assertory, universal, and real.

2. "The three angles of a triangle are together equal to two right angles": categorical, affirmative, assertory in form, but really necessary, universal, and real.

3. "Some elements are not metals": categorical, negative, assertory, particular, and real.

4. "None but material bodies have weight": this proposition really means that "all things having weight are material bodies." In this form it is an A proposition. In the original form, it may be regarded as an E proposition, "no not-material bodies have weight," signifying that having weight is denied of all things other than, or except, material bodies, that none that have weight are other than material bodies, and this last is the same as "all things having weight are material bodies," the proposition we have substituted above for the original one. It should be noted that the proposition does not mean that every material body has weight.

5. "All metals except mercury are solids."—In this proposition 'solids' is affirmed of all metals except mercury, and the proposition may, therefore, be regarded as an A proposition and described as categorical, affirmative, assertory, universal, and real. Or it may be taken as an I proposition, 'some metals are solids," but in this degraded form, the full meaning of the original proposition is not expressed. Or we might state the names of all the metals except mercury, and form a proposition with them all as the subject and 'solids' as the predicate as before. For example, 'gold, copper, iron, silver, &c., are solids.' Such a proposition would be a combination of the several propositions, having each a certain metal for its subject, and 'is a solid' for its copula and predicate. Thus, 'gold is a solid,' 'copper is a solid,' 'iron is a solid,' and so forth.

6. "All is not gold that glitters," = "All that glitters is not gold." This proposition is really O, though it has the form of E. It really means that at least some thing that glitters is not gold.

7. "If mercury be heated, it will expand": conditional, affirmative, assertory, universal, real.

8. "All men are rational, but all are not wise": this sentence is a combination of the two propositions—(1) 'All men are rational' (A), and (2) 'All men are not wise' (O).

9. "Gravity as well as heat can produce motion": a combination of the two propositions, (1) 'Gravity can produce motion' (A), and (2) 'Heat can produce motion' (A).

Examples for Solution.

Treat the propositions[1] given below as follows:—

I.—Describe the logical characters of each of them.

II.—Give the contradictory, the contrary or subcontrary, and the subalternant or subalternate of each of them.

III.—State the relation of the predicate to the subject in each of the affirmative propositions.

IV.—In the case of a disjunctive proposition, state the hypothetical propositions, one or other of which is equivalent to it.

1. Every pure substance consists of similar molecules.
2. Some animals have no power of locomotion.
3. Sensations are passive states of the mind.
4. Nothing is annihilated.
5. All metals except one are solid.
6. Benevolence is a virtue.
7. Only the virtuous are happy.
8. Certain metals are ductile.
9. Some substances have no cause.
10. Uneasy rests the head that wears a crown.

[1] Most of the propositions given here are taken from Ganot's *Popular Natural Philosophy*, Roscoe's *Chemistry*, and Reid's *Inquiry*, exactly in the form in which they are expressed by the authors. They are kept in that form in order that students may acquire the habit of describing the characters of propositions as they actually occur in the works of authors, instead of the contracted and artificial propositions of the Logician.

11. None were there.
12. None but sensations can resemble sensations.
13. Metals conduct heat and electricity.
14. Oxygen is a colourless, invisible gas, possessing neither taste nor smell.
15. Hydrogen is the lightest body known.
16. Matter is indestructible.
17. Most of the acids are soluble in water.
18. All acids contain hydrogen and always contain also oxygen.
19. The passage of water to the state of ice, and the return of the latter to the liquid state, are physical phenomena.
20. The mass of a body is the quantity of matter contained in the body.
21. The elementary atoms can unite with each other to form compounds, but cannot be destroyed by any known process.
22. If molecular attraction were the only force acting upon the small particles of which bodies are composed, they would come into complete contact.
23. All bodies are extended, impenetrable, divisible, porous, compressible, and elastic.
24. Strictly speaking, impenetrability only applies to the atoms of bodies.
25. Divisibility, porosity, compressibility, and elasticity do not apply to atoms, but only to bodies or aggregates of atoms.
26. Two portions of matter cannot simultaneously occupy the same portion of space.
27. Compressibility is both a consequence and a proof of porosity.
28. Both rest and motion are either absolute or relative.
29. Bodies are either opaque or transparent.
30. If a small quantity of manganese di-oxide be mixed with the potassium chlorate, the oxygen is given off from the chlorate at a much lower temperature.
31. Oxygen can be prepared by heating powdered potassium chlorate in a small thin glass flask.
32. All the elements with the single exception of fluorine combine with oxygen to form oxides.
33. Sulphur exists in three modifications.
34. Many organic bodies are completely decomposed and charred by strong sulphuric acid.

35. Phosphorus does not dissolve in water, alcohol, or ether.
36. Arsenic is sometimes found in the free state, but more frequently combined chiefly with iron, nickel, cobalt, and sulphur.
37. Truly these ideas seem to be very capricious in their agreements and disagreements.
38. Motion is either rectilinear or curvilinear.
39. Each kind of motion is either uniform or varied.
40. Matter cannot change its own state of motion or of rest.
41. A power is a force which tends to produce motion.
42. The surfaces of bodies are never perfectly smooth.
43. Without friction on the ground neither man nor animals, neither ordinary carriages nor railway ones, could move.
44. If all impeding causes were removed, a body once in motion would continue to move for ever.
45. Some brutes are sensible of honor and disgrace.
46. Hardness and softness are neither sensations, nor like any sensations.
47. A sensation can only be in a sentient being.
48. No man can conceive any sensation to resemble any known qualities of bodies.
49. If we trust to the conjectures of men of great genius in the operation of nature, we have only the chance of going wrong in an ingenuous manner.
50. If dry chlorine gas be passed over silver nitrate, silver chloride is formed, oxygen is given off, and a white crystalline substance produced, which, on analysis, is found to be nitrogen peroxide.
51. If nitrogen monoxide gas (or laughing gas) be brought under a pressure of about 30 atmospheres at 0° C. or if it be cooled down to −86° C. under the ordinary pressure, it forms a colourless liquid.
52. If this liquid be cooled below −115° C., it solidifies to a transparent mass.
53. If carbon were not present in the earth, no single vegetable or animal body such as we know could exist.
54. If a piece of lime be held in the oxyhydrogen flame, it becomes strongly heated and gives off intense light.

55. The ignition of phosphorus takes place by slight friction, or by a blow, and even the heat of the hand may cause this substance to ignite.
56. The number of the metals is much larger than that of the non-metals.
57. The atmosphere is the gaseous envelope encircling the earth.
58. If a series of electric discharges be passed through pure oxygen, the gas becomes diminished in volume by about one-twelfth, and is partly transformed into ozone.
59. If we would know the works of God, we must consult themselves with attention and humility.
60. I know that I know.
61. Consciousness is an actual and not a potential knowledge.
62. If mediate knowledge be in propriety a knowledge, consciousness is not co-extensive with knowledge.
63. Where two, three, or more mental states are confounded, we are conscious of them as one.
64. Without memory our mental states could not be held fast, compared, distinguished from each other, and referred to self.
65. The theory of ideas is, indeed, very ancient, and hath been very universally received.
66. Common sense holds nothing of philosophy, nor needs her aid.
67. To attend accurately to the operations of our mind, and make them an object of thought, is no easy matter, even to the contemplative, and to the bulk of mankind is next to impossible.
68. He must either be a fool, or want to make a fool of me, that would reason me out of my reason and senses.
69. If philosophy contradicts herself, befools her votaries, and deprives them of every object worthy to be pursued or enjoyed, let her be sent to the infernal regions from which she must have had her origin.
70. To reason against any of these kinds of evidence is absurd, nay to reason for them is absurd.
71. We must either admit the conclusion or call in question the premises.
72. Ideas seem to have something in their nature unfriendly to other existences.

73. If one set of ideas makes a covenant, another breaks it, and a third is punished for it, there is reason to think that justice is no natural virtue in the ideal system.
74. The smell of a rose is a certain affection or feeling of the mind.
75. Some tastes and smells stimulate the nerves and raise the spirit.
76. That such a noise is in the street, such another in the room about me; that this is a knock at my door, that a person walking upstairs,—is probably learned by experience.
77. The parallelism of the eyes in general is the work of nature.
78. If a man hath lost the sight of one eye, he very often loses the habit of directing it exactly to the object he looks at.
79. A miniature painter or an engraver sees very near objects better than a sailor.
80. That we see objects single with two eyes, as well as that we see objects erect by inverted images, is attributed by Bishop Berkeley and Dr Smith entirely to custom.
81. If two visible appearances have the same visible place, they are incapable of distinction, and we see the objects single or one object only.
82. A just interpretation of nature is the only sound and orthodox philosophy.

CHAPTER II.

THE THEORY OF PREDICATION AND THE IMPORT OF PROPOSITIONS.

§ 1. WHAT is the import or meaning of a proposition or predication? What is the thought or fact expressed by it? What is the signification of its subject, of its predicate, and of its copula? In other words, in all propositions or predications of the type "A is B" (or "A is not B"), what is A, what is B, and what is the relation between them? A consistent answer to this question is a theory of Predication and of the import of Propositions. On this most important subject, there is great difference of opinion among logicians. It is proposed to give here an account of their views, as far as possible, in their own language and from their own point of view.

§ 2. I. The natural view seems to be that 'B' is an attribute, and that this attribute is referred or said to belong to the objects denoted by 'A,' as in the proposition 'Snow is white,' 'whiteness' is said to belong to the thing called 'snow.' This view is thus explained and defended by Dr James Martineau: "In saying 'Birds are warm-blooded,' we neither think of class within class, nor of attribute within attribute: the word 'warm-blooded' represents to us no conception of a *genus;* it is not a name, but a mere attributive. The word 'birds' expresses to us *no attribute*, as such; it is not a mere attributive, but a name. The term in the predicate acts upon the mind by its connotation, or in its comprehension; the term in the subject, by its denotation or in its extension; and the foregoing sentence has its

import in this,—that we refer the attribute 'warm-blood' to the class of objects 'birds.' Hence it is that, while a purely connotative word (an adjective) is all that is required in the predicate, a denotative term is indispensable in the subject.......The mind predicates nothing except about substantive objects of thought; and of them (in the class of propositions now under consideration) it predicates nothing but attributes[1]." According to Dr Martineau, the Denotative or Class Theory of Predication and Mill's Connotative Theory are both psychologically false.

All propositions do not, according to Dr Martineau, express the relation of substance and attribute. There are classes of propositions which express other relations. "The notion of substance and attribute, with the relations of genera and species to which it introduces us, is but one......of several categories of thought." "It is the basis of all class-reasoning, and supplies the common logical canon of necessity, that 'what is true of the containing is true of the contained.'" But all Demonstrative Reasoning should not be forced into this single type. There are other types of Demonstrative Reasoning founded upon other relations expressed by propositions. Propositions may, for example, express the relations of time and space, of cause and effect, of resemblance and difference, and give rise to types of Demonstrative Reasoning quite distinct from that of class-reasoning. "The attempt," says Martineau, "to coerce all reasoning into this single type—comprehensive as it is—appears to us arbitrary in itself,—and precluded from success except on condition of much violent psychology. The ideas of space and time, of cause and effect, of resemblance and difference, seem to involve distinct laws of thought, to create for themselves special elements and functions of language, and to require separate canons of Logic."

According to Martineau, therefore, there are different classes of propositions expressing different categories of thought, and there are as many distinct types of Demonstrative Reasoning as

[1] *Essays*, Vol. II. p. 351.

there are fundamental laws of thought arising from these categories.

§ 3. II. Hamilton's view:—

"To judge is to recognize the relation of congruence or of confliction, in which two concepts, two individual things, or a concept and an individual, compared together, stand to each other. This recognition considered as an internal consciousness, is called a Judgment, considered as expressed in language, it is called a Proposition or Predication." This definition is then explained. "When two or more thoughts are given in consciousness, there is in general an endeavour on our part to discover in them and to develop a relation of congruence or of confliction, that is, we endeavour to find out whether these thoughts will or will not coincide,—may or may not be blended into one; if they coincide, we judge, we enounce their congruence or compatibility: if they do not coincide, we judge, we enounce their confliction or incompatibility. Thus, if we compare the thoughts, water, iron, and rusting, we find them congruent, and connect them into a single thought, thus, water rusts iron; in that case we form a judgment[1]." Hamilton finally defines a judgment as follows: "We may, therefore, articulately define a judgment or proposition to be the product of that act in which we pronounce that of two notions thought as subject and as predicate, *the one does or does not constitute a part of the other*, either in the quantity of extension, or in the quantity of comprehension[2]."

According to Hamilton, therefore, 'A' and 'B' in the typical judgment 'A is B' are two concepts, the one forming a part of the other. From what he says elsewhere, we know he maintains that in the quantity of comprehension, 'B' is a part of 'A,' and that in the quantity of extension, 'A' is a part of 'B.' That is, the proposition has a two-fold meaning according as you take the two concepts 'A' and 'B' in their comprehension or in their extension. When 'A' and 'B' are taken in their comprehension, the meaning of the proposition is that the elementary notions constituting the concept 'B' are a part of those constituting the

[1] Hamilton's *Lectures*, Vol. III. pp. 226—7. [2] *Ibid.* p. 229.

concept 'A'; and when they are taken in extension, the meaning is that the individual things or objects included in the extension of 'A' are a part of those included in the extension of 'B.'

§ 4. III. Mansel's view:—

"When I assert that A is B, I do not mean that the attributes constituting the concept A are identical with those constituting the concept B; for this is only true in identical judgments; but that the object in which the one set of attributes is found is the same as that in which the other set is found." For example, "when I assert that the rose is fragrant, I imply that the thing which affects in a certain manner my power of sight, is in some manner identical with that which affects in a certain way my power of smell." Mansel thus defines a concept and a judgment: "A concept is a collection of attributes united by a sign, and representing a possible object of intuition." "A judgment is a combination of two concepts, related to one or more common objects of possible intuition." "The subjects of all logical judgments which are to be distinguished from the psychological, such as the spontaneous judgments of perceptive and imaginative faculties, are concepts[1]."

According to Mansel, therefore, 'A' and 'B' are both concepts, and the meaning of the proposition (when not identical) is that the attributes signified by both 'A' and 'B' exist in the same object or objects.

§ 5. IV. Ueberweg's view:—

"The judgment is the consciousness of the objective validity of a subjective union of conceptions, whose forms are different, but belong to each other. It is the consciousness, whether or not the analogous combination exists between the corresponding objective elements. As the individual conception corresponds to the individual existence, so the judgment in its various forms corresponds to, and is the subjective copy of, the various objective relations. A judgment expressed in words is an assertion or proposition[2]."

[1] *Prolegomena Logica*, 2nd edition, 1860, pp. 67—69.

[2] Ueberweg's *Logic*, p. 187.

According to Ueberweg, therefore, 'A' and 'B' are two conceptions or concepts, and the meaning of the judgment 'A is B' is that, corresponding to the union of the two concepts, there is an objective union. In other words, a mere combination of conceptions is not a judgment; but there must be the conviction that the combination has objective validity.

§ 6. V. Mill thus states the problem to be solved:—

"We have, then, to inquire, on the present occasion, not into judgment, but judgments; not into the act of believing, but into the thing believed. What is the immediate object of belief in a proposition? What is the matter of fact signified by it? What is it to which, when I assert the proposition, I give my assent, and I call upon others to give theirs? What is that which is expressed by the form of discourse called a proposition, and the conformity of which to fact constitutes the truth of the proposition[1]?"

§ 7. Mill declares at the outset that a proposition is not about our ideas or concepts of things, but about things themselves, and dismisses all the theories of predication which have our ideas or concepts for the subject and the predicate of the proposition, with the remark that "the notion that what is of primary importance to the logician in a proposition is the relation between the two *ideas* corresponding to the subject and predicate (instead of the relation between the two phenomena which they respectively express) seems to me one of the most fatal errors ever introduced into the philosophy of Logic, and the principal cause why the theory of the science has made such inconsiderable progress during the last two centuries[2]." He then points out that Hobbes's theory that a predicate is a name of that of which the subject is a name, is a sufficient account when 'A' and 'B' are both proper names, but that it is inadequate for all propositions whose subject and predicate are not proper names, because it entirely overlooks the meaning of names in connotation.

[1] Mill's *Logic*, Vol. I. p. 99. [2] *Ibid.* p. 98.

§ 8. Mill then shows that the Denotative or Class Theory of Predication accordingly to which predication consists in referring something to a class, *i.e.*, in placing an individual under a class or one class under another, is hardly better than the theory of Hobbes. "There is," says he, "no real difference, except in language, between this theory of predication and the theory of Hobbes. For a class is absolutely nothing but an indefinite number of individuals denoted by a general name. The name given to them in common is what makes them a class. To refer anything to a class, therefore, is to look upon it as one of the things which are called by that common name. To exclude it from a class, is to say that the common name is not applicable to it[1]." The Class Theory of Predication is, argues Mill, moreover psychologically false. For in the proposition 'snow is white,' I am not thinking of 'white objects' as a class, but only of 'snow' as an object and the sensation of 'white' which it gives me.

§ 9. A view that is closely connected with the Denotative or Class Theory of Predication, and is, in fact, only a special development of it, is the equational view of propositions. According to this view, the proposition 'A is B' is an equation, 'A' and 'B' corresponding to the two sides of the equation, and 'is' to the sign of equality between them; and the meaning of the proposition is that the things denoted by 'A' are identical with those denoted by 'B.' This view is adopted by Hamilton in his later writings. It is the direct consequence of the doctrine of the Quantification of the Predicate. This doctrine is, that in thought the quantity of the predicate as well as that of the subject is implicitly contained, and that, according to the principle, that "Logic postulates to be allowed to state explicitly in language all that is implicitly contained in the thought," it may be expressed by such words as 'some,' 'all,' &c., before the predicate.

Adopting this doctrine, Hamilton obtains the following eight

[1] Mill's *Logic*, Vol. I. p. 104.

forms of propositions instead of the four we have given in a previous chapter:—

(1) All A is some B. (A.)
(2) All A is all B. (U.)
(3) No A is any B. (E.)
(4) No A is some B. (η.)
(5) Some A is some B. (I.)
(6) Some A is all B. (Y.)
(7) Some A is not any B. (O.)
(8) Some A is not some B. (ω.)

Mill objects to the adoption of the above view on the following grounds[1]:—(1) The theory is psychologically false, because the predicate of a proposition is not thought of in its extension, but only in its comprehension. In the proposition "all oxen ruminate," nobody thinks of other ruminating animals, and none ever asks the question whether or not there are other animals that ruminate; all that anyone is thinking of is the phenomenon or attribute of ruminating in reference to 'oxen.' (2) All reasoning being carried on in the ordinary forms of expression, it is desirable that every proposition in logical form should be the exact equivalent of some proposition in the common form. On this ground the proposition "all A is all B" is inadmissible, because there are none corresponding to it in ordinary language, because it is really a compound of two ordinary propositions, *viz.*, "all A is B" and "all B is A"; since it can never be accepted without proving these two. Similarly, if you take "some A is B" to mean "some A is some B only," you not only change the real logical meaning of 'some' as meaning 'not none,' it may be 'all,' into 'a part only,' 'not the whole,' but you make the proposition "some A is some B" really a double judgment, an implicit expression of the two explicit judgments, *viz.*, "some A is some B" and "some other A is not any B." (3) Logic should start with the simplest or most elementary judgments. But "all A is all B," "some A is

[1] Mill's *Examination of Hamilton's Philosophy*, Chap. XXII.

some B" are complex, consisting of two as we have just seen, while "A is B" is the simplest and most elementary, than which there cannot be any simpler.

Hamilton anticipates some of Mill's objections. He says:— But, in fact, ordinary language quantifies the predicate so often as this determination becomes of the smallest import. This it does either directly, by adding *all*, *some*, or their equivalent predesignations, to the predicate; or it accomplishes the same end indirectly, in an exceptive or limitative form. (*a*) Directly,— as "Peter, John, James, &c., are *all* the Apostles," "Mercury, Venus, &c., are *all* the planets." (*b*) But this is more frequently accomplished indirectly, by the equipollent forms of limitation or inclusion, and exception. For example, by the limitative designations, *alone* or *only*, we say, "God alone is good," which is equivalent to saying, God is all good, that is, God is all that is good; "Virtue is the only nobility," that is, virtue is all noble, that is, all that is noble. "Faith, hope, charity, alone justify." "Of animals man alone is rational," that is, man is all rational animal. "What is rational is alone or only risible," that is, "all rational is all risible, &c." Of the exceptive form Hamilton gives the following examples:—"On earth there is nothing great but man," which means "Man is all earthly great." "In man there is nothing great but mind," which means "Mind is all humanly great," that is, "all that is great in man[1]."

[1] The following note by Hamilton on the import of what are called exclusive and exceptive particles is worth quoting:—They are, "one, only, alone, exclusively, precisely, just, sole, solely; nothing but— not—except, beyond. (1) These particles annexed to the subject predesignate the predicate universally, or to its whole extent, denying its particularity or indefinitude, and definitely limiting it to the subject alone; as, 'man alone philosophises,' 'the dog alone barks,' 'man only is rational,' 'of material things there is nothing living (but) not organized, and nothing organized not living,' 'God alone is to be worshipped,' 'some men only are elect.' (2) Annexed to the predicate, they limit the subject to the predicate, but do not define its quantity, or exclude it from other subjects; as, 'Peter only plays,' 'the sacra-

"The non-quantification of the predicate in thought," argues Hamilton, "is given up by the logicians themselves, but only in certain cases where they were forced to admit, and to the amount which they could not possibly deny. The predicate, they confess, is quantified by particularity in affirmative, by universality in negative, propositions. But why the quantification, formal quantification, should be thus restricted in thought, they furnish us with no valid reason[1]."

§ 10. Mill's own theory, which may be called the Connotative or Attributive Theory of Predication, is that the proposition 'A is B' expresses a certain relation between the attributes connoted by 'A' and 'B' respectively,—or, more properly, a certain connection or relation between the phenomena on which the attributes are respectively founded and through which they are known,—and that the relation expressed by it is that of co-existence, succession, causation, resemblance, or mere existence[2]. Take, for example, the proposition "All men are mortal":

ments are only two,' 'the categories are only ten,' 'John drinks only water.' (3) Sometimes the particles sole, solely, single, alone, only, &c., are annexed to the predicate as a predesignation tantamount to '*all*'; as, 'God is the single,—one,—alone,—only,—exclusive,—adequate,—object of worship.'"

[1] Hamilton's *Lectures*, Vol. IV. pp. 261—5.

[2] In the case of a proposition whose subject is a proper name and has, therefore, according to Mill, no signification in connotation, the meaning of the proposition, according to him, is, that the attribute or attributes connoted by the predicate belong to the individual thing denoted by the subject. For example, the proposition "Socrates is a philosopher" means that the attributes of being a philosopher belong to the individual denoted by the proper name Socrates. If both the subject and the predicate of a proposition are proper names, then, according to Mill, Hobbes's theory is a sufficient account of it: as examples of such propositions he gives:—'Tully is Cicero,' 'Hyde was Clarendon,' &c., the whole meaning of such propositions is, that the predicate is a name or meaningless mark for the same thing for which the subject is a mark.

its meaning is that the objects denoted by the subject possess the attributes connoted by the predicate. The objects are not, however, individually designated. "They are pointed out only by some of their attributes; they are the objects called '*men*,' that is possessing the attributes connoted by the term 'man,' and the only thing known of them may be these attributes; indeed the proposition is general, and the objects denoted by the subject are, therefore, indefinite in number, most of them are not known individually at all. The assertion is, * * therefore, that the attributes which the predicate connotes are possessed by each and every individual possessing certain other attributes, that whatever has the attributes connoted by the subject has also those connoted by the predicate, that the latter set of attributes constantly accompanies the former set. Whatever has the attributes of man has the attribute of mortality; mortality constantly accompanies the attributes of man[1]."

To the objection that we naturally construe the subject of a proposition in its extension, and the predicate in its intention, Mill replies that "though it is true that we naturally construe the subject of a proposition in its extension, this extension, or, in other words, the extent of the class denoted by the name is not apprehended or indicated directly, and that it is both apprehended and indicated solely through the attributes."

But what is an attribute? "Every attribute," says Mr Mill, "is grounded on some fact or phenomenon, either of outward sense or of inward consciousness; and to possess an attribute is another phrase for being the cause of, or forming part of, the fact or phenomenon upon which the attribute is grounded[2]." The proposition 'All men are mortal,' therefore, really means that "wherever the various physical and mental phenomena on which the attributes of 'man' are grounded are all found, there we have assurance that the other physical and mental phenomenon, called death, will not fail to take place. The proposition does not affirm *when;* for the connotation of the word 'mortal' goes

[1] Mill's *Logic*, Vol. I. p. 109. [2] *Ibid*. p. 109.

no farther than to the occurrence of the phenomenon at some time or other, leaving the particular time undecided[1]." The relation asserted here between the two sets of phenomena is one of either co-existence or succession. Similarly in the propositions 'A generous person is worthy of honor,' 'Thoughtlessness is dangerous,' 'Prudence is a virtue,' the relation expressed is co-existence or succession, and the things between which the relation exists are the attributes connoted or signified by the subject and the predicate of the proposition, or rather the phenomena and actions upon which they are grounded.

Besides co-existence and sequence propositions may express causation or mere existence, as in the case of nöumena, or resemblance, as in such propositions as this, 'The heat of to-day is equal to the heat of yesterday.' These relations are expressed not only between phenomena, but also between nöumena, and between phenomena and nöumena. The relation of causation is only provisionally recognized, subject to the analysis of it under the head of causation.

Mill thus sums up the result of his investigation:—

"Existence, co-existence, sequence, causation, resemblance, one or other of these is asserted or denied in every proposition which is not merely verbal. This five-fold classification is an exhaustive classification of matters of fact, of all things that can be believed or tendered for belief; of all questions that can be propounded and all answers that can be returned to them[2]." On the suggestion of Professor Bain that co-existence is of two kinds,—one in different places at the same time, and the other in the same part or place, as the co-existence or co-inherence in every atom of gold, of the attributes of a certain specific gravity, tenacity, fusibility, lustre, colour, &c., Mill divides all co-existence and succession into Order in Time and Order in Place, the former including Bain's Coinhering Attributes. Of the five classes given by Mill, Bain adopts only three:—(1) Co-existence, (2) Succession, including Causation, (3) Equality or Inequality.

[1] *Logic*, Vol. I. p. 110. [2] *Ibid.* p. 116.

§ 11. A few remarks on Mill's Theory:—

The first remark to be made on Mill's theory is, that he does not show, either deductively or inductively, either from the nature of relations or from an enumeration of them, that his five-fold classification is an exhaustive one; that every possible relation between attributes has been included in his list.

The second remark is, that Mill does not give a sufficient account of the meaning of those propositions which he calls verbal. By calling them verbal, a name not without a touch of contempt, he seems to consider them as of no importance. But they are as important as those which he calls real propositions. Kant calls the two classes analytical and synthetical, respectively, and these two terms seem to express the distinction between them much better than Mill's names. What is the meaning of a verbal proposition even on Mill's own theory? It is that the connotation of the predicate is a part of the connotation of the subject, that is, the phenomena on which the attribute signified by the predicate is grounded are a part of the phenomena on which the attributes connoted by the subject are grounded. The meaning of the proposition 'Man is rational,' for example, is that the phenomena on which the attribute, rationality, is grounded are a part of, or included in, the phenomena on which the attributes signified by the term 'man' are grounded. Thus it would seem, that, to the five heads given by Mill, a sixth, namely, inclusion or containing of attributes, should be added. This last is different from any that are mentioned by Mill. It is not the same as co-existence, for two phenomena or attributes may co-exist without one forming a part of the other. Thus gravity and inertia co-exist, but one is not contained in the other; while animality is contained in humanity. A verbal proposition does not merely explain the meaning of a name, but expresses, like a real proposition, a relation between phenomena or attributes. The relation expressed by it is that of containing or inclusion. The different relations between phenomena or attributes may be thus shown in a tabular view:—

RELATIONS,

Expressed by Analytical propositions.	Expressed by Synthetical propositions.
	Co-existence, Sequence, Causation, Existence, Resemblance.

When both the subject and the predicate of a proposition are taken in intension, Hamilton seems to recognize only one relation between them, namely, the relation of containing or inclusion; and this he does by enlarging the intension of the subject, that is, by putting into it all that is known of the thing or things denoted by the subject. Thus, according to him, all judgments are analytical or verbal, the attribute signified by the predicate being a part of the intension of the subject. He says in the *Lectures*, Vol. II. p. 336, quoting with approval from Crousary:—

"Every time we judge, we compare a total conception with a partial, and we recognize that the latter really constitutes a part of the former." Again, "when we judge, we must have, in the first place, at least two notions; in the second place we compare these; in the third, we recognize that the one contains or excludes the other; and in the fourth, we acquiesce in the recognition." "When I say, 'body is divisible,' among the notions which occur in forming my conception of body, I particularly attend to that of 'divisible,' and finding that it really agrees with the others, I judge accordingly that the body is divisible."

Another remark suggested by Mill's theory is, that it makes the meaning of a proposition depend upon what is more or less variable, indefinite, and uncertain. Take, for example, the proposition 'Man is mortal.' According to Mill its meaning is that 'mortality' co-exists with 'humanity,' that whatever has the attribute 'humanity' has the attribute 'mortality.' Now, what is meant by 'humanity'? What are the essential elements of it? Is it possible to give a final definition of it? If not, how am I to know what does and what does not possess it? Again,

the connotation of the term 'man' is not the same to all persons, being different to different classes according to the kind and degree of their education and experience. Nor is it anything constant and fixed. On the contrary, it must vary with the progress in our knowledge of man in all his aspects. Or take the proposition 'All material bodies gravitate.' Its meaning, according to Mill, is that whatever has the attribute of a 'material body' has also the attribute of 'gravitating.' Now, what are the attributes of a material body? How am I to know whether a particular body is material or not? Is the luminiferous ether (the medium of light), for example, material? Thus the connotation of terms being variable and uncertain, the meaning of a proposition, on Mill's theory, must partake of its uncertainty, variability, and indefiniteness.

The last remark that I will make on Mill's theory is connected with the import or real meaning of a term, and should, perhaps, have been made first. In the chapter on Terms, Mill says that a common or general term directly signifies objects or things, and implies or indirectly signifies attributes; so the connotation of a term is taken in that chapter to be its implied or indirect meaning, and its denotation the direct or explicit meaning[1]. But in his theory of the Proposition, the former is taken as the direct or essential meaning, while the latter is entirely passed over. Consistency seems to require that Mill should have regarded the connotative or rather attributive meaning of a term as its direct and explicit meaning, and the denotative meaning as indirect and implicit.

§ 12. From what we have given above of the views of Logicians, it is evident that they differ (1) as to the relation of A and B (subject and predicate) and (2) as to the way in which A and B are to be interpreted (that is, the meaning of subject and predicate).

[1] Mill's *Logic*, Vol. I. pp. 31, 32.—"A connotative term is one which denotes a subject, and implies an attribute," p. 31. Again, "The name is, therefore, said to signify the subjects directly, the attributes indirectly, &c.," p. 32.

As regards the first point, Hamilton, for instance, recognizes the relation of containing or not-containing (inclusion or exclusion) either in the quantity of extension or in the quantity of comprehension, arising from the 'relation of congruence or confliction.' Mansel holds that the two sets of attributes expressed by A and B must be capable of existing together in some possible object of intuition, that is, the relation of A and B is that of compatibility or incompatibility. According to Ueberweg the relation of A and B must correspond to an objective relation, that is, to a relation really existing among things. Martineau recognizes the relation of substance and attribute, and, also, the relations of time and space, of cause and effect, and of resemblance and difference. Mill gives the relations expressed by all propositions under five heads: (1) Existence, (2) Co-existence, (3) Succession, (4) Causation, (5) Resemblance. Bain includes all under three classes, (1) Co-existence, (2) Succession, (3) Equality or Inequality.

The different views arising from difference on the second point, namely, the way in which A and B are interpreted by Logicians, may be noted as follows:—(1) The Ordinary or Predicative View in which A is taken in denotation (or extension) and B in connotation (or comprehension), and the relation of A and B is that of subject and attribute. "The light," says Dr Venn, "in which a proposition has to be consistently interpreted on this view is that of *predication*. We distinguish between subject and attribute here, and we assert that a given subject does or does not possess certain attributes[1]." Of the four forms A, E, I, O, arising from this view of propositions, Dr Venn says, "These forms appear to be naturally determined by the ordinary needs of mankind, and the ordinary pre-logical modes of expressing those needs; all that Logic has done being to make them somewhat more precise in their signification than they conventionally are[2]." Again, "As just remarked, these forms of proposition certainly seem to represent the most primitive and natural

[1] *Symbolic Logic*, p. 3. [2] *Ibid.* p. 3.

modes in which thought begins to express itself with accuracy[1]."

According to this view, all relations expressed by propositions may be reduced to the single type of the relation of subject and attribute. The subject of a proposition may be anything that can possess an attribute or attributes. It may be a substance, a phenomenon, or an attribute. The predicate of a proposition is an attribute; and even when the predicate is a concrete term, the term is interpreted in its connotation (or comprehension).

This view of Propositions does not ignore the relations of space and time, of cause and effect, of resemblance and difference, expressed by many propositions; but it holds that, for logical purposes, they may all be reduced to the relation of subject and attribute. Some Logicians holding this view so far as a certain class of propositions, namely, those expressing the relation of substance and attribute, are concerned, maintain that the other relations, such as those of time and space, of cause and effect, of resemblance and difference, can not, or should not, be reduced to the single type of subject and attribute. According to them, there are different classes of propositions founded upon different categories of thought and giving rise to distinct types of rea-

[1] *Symbolic Logic*, p. 4.

[2] The relation of subject and attribute is also called the relation of substance and attribute. It is not necessary to inquire here into the nature of this relation, or into the meaning of Subject, Substance, Thing, or Attribute, or to discuss the question as to whether an attribute possessing attributes becomes a substance (or thing), or remains an attribute. For the Predicative view, it is sufficient if propositions expressing other relations can, in some way, be understood to express the relation of subject and attribute; and this may be done in the following manner:—The proposition "A is equal to B," for example, expressing the relation of Equality, means, according to this view, that the attribute of being equal to B is possessed by A, whether A and B be things or attributes; the proposition "A is the cause of B," expressing the relation of Cause and Effect, means, according to this

(2) The Denotative View, in which both A and B are taken in denotation (or extension). This view includes (*a*) Hobbes' View, (*b*) the Class View, in which the class or group of things denoted by A is included in the class or group of things denoted by B, and (*c*) the Equational View, in which the things denoted by A are the same as those denoted by B.

(3) The Connotative or Attributive View, in which both A and B are taken in connotation, and the relation expressed by the proposition is variable and depends on the nature of A and B. It does not reduce all relations expressed by propositions to one single type. It recognizes different fundamental relations, and distinct modes of reasoning arising from those relations. Thus, according to this view, there are Mathematical reasonings founded upon the relations of Equality or Inequality, Inductive reasonings founded upon the relations of succession or of cause and effect, besides Reasonings which are founded upon the relation of co-existence of attributes, which takes the place of the relation of substance and attribute.

(4) The Denotative-Connotative View, in which A and B are taken both in denotation (or extension) and in connotation (or comprehension), and the relation of A and B is a twofold one. Hamilton, for instance, holds that when both A and B are taken in extension, A is contained in B, and that when both A and B are taken in comprehension, B is contained in A.

There is another point on which Logicians differ in their views of the Proposition. It is connected with the different views which they take of Logic as a science. The different views of the Proposition arising from difference on this point may be noted as follows:—

(1) The Conceptualist or Subjective View, in which both A and B are concepts, not necessarily corresponding to really existing things, but true of possible things, that is, of things that may be realised in Thought.

view, that the attribute of being the cause of B is possessed by A whatever A and B may be.

(2) The Materialist or Objective View, in which both A and B are concepts corresponding to really existing things, and the relation of A and B is a relation of concepts corresponding to a relation of things: e.g. Ueberweg's view.

(3) There is another view which is usually identified with the second view, but which should be distinguished from it. I mean the view according to which A and B stand for really existing things, and the relation of A and B is a relation of things: e.g. Spencer's view.

Mill, in his *Examination of Hamilton's Philosophy*, holds the second view; but in his *System of Logic* he very nearly gives it up and passes on to the third view. Among English Logicians he seems to occupy an intermediate position between subjective or conceptualist Logicians, represented by Hamilton and Mansel, and objective Logicians, represented by Mr Spencer and Mr Carveth Read.

The difference between the second and the third view, is that, according to the former, the two terms of a proposition are two concepts corresponding to really existing things, while, according to the latter, the two terms are really existing things or phenomena themselves. The upholders of the third view do not seem to face the question as to how things or phenomena can be either the subject or the predicate of proposition, without being thought, that is, without being concepts. The upholders of the second view recognize this necessity and treat in Logic of the forms and relations of Thought as corresponding to the forms and relations of Things, while the upholders of the third view profess to treat of the forms and relations of things themselves[1].

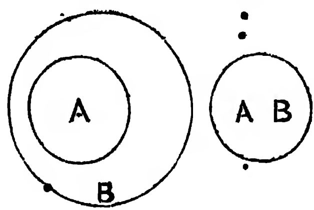
[1] See Appendix E, "The Nature and Province of Objective Logic."

CHAPTER III.

The Meaning and Representation of A, E, I, O by Diagrams.

§ 1. A stands for any Universal Affirmative proposition of the type 'All A is B.' It may be represented by the two diagrams, A, 1, and A, 2. According to the ordinary or predicative view of propositions, the meaning of A is that the attribute connoted by 'B' belongs to all the things or objects denoted by 'A,' and the implication is that it may or may not belong to any other things. The diagrams represent this, thus,—the circle A stands for the things denoted by the term A, and the circle B for the cases in which the attribute connoted by the term B occurs; the first diagram shows that those cases are more numerous than the things, and the second shows that the two are equal. The meaning of the proposition will be represented by one or other of the two diagrams.

A. 1. A, 2.



According to the denotative view of propositions, the meaning of A is that the whole of the class denoted by the term A is included in the class denoted by the term B, or that the former is co-extensive with the latter. And this is shown by the diagrams,—in the first, the whole of the class A is a part of the class B, and in the second, the two classes coincide. The mean-

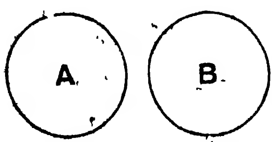
ing of the proposition will be represented by one or other of the two diagrams.

According to the connotative view of propositions, the meaning of A is that the attribute connoted by 'B' accompanies the attribute connoted by 'A' in every case, that is, wherever the latter is, there the former is. The diagrams may be understood to represent this, thus,—the first shows that the cases in which the attribute connoted by A occurs are a part of, or are less numerous than, the cases in which the attribute connoted by B occurs; the second shows that the two classes of cases coincide or are equal in number.

Thus, on all the three views, A can be represented by these two diagrams. On each of them, the subject of A is always taken in its whole extent, while the predicate is always taken in a partial and sometimes also in its total extent. This is plainly the case on the first and second views. On the third, too, this is the case, because in all cases the attribute connoted by A is accompanied by the attribute connoted by B. This fact is what is meant by saying that, *in an A proposition, the subject is distributed, and the predicate undistributed.* By the extent of an attribute is meant the number of cases in which it occurs.

§ 2. E stands for any Universal Negative proposition of the type 'No A is B.' It is represented by the following diagram. The meaning of the diagram is different on the different views of propositions.

E.



On the first view, the circle A stands for the things denoted by the term A; and the circle B for the cases in which the attribute connoted by the term B occurs; and the diagram shows that the one set is quite distinct from the other,—that the attribute connoted by B does not in any case belong to any of the things denoted by A.

On the second view, the two circles A, B stand for two classes denoted respectively by A and B; and the diagram shows that

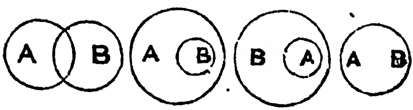
the one class is entirely excluded from the other, that the things denoted by B are quite distinct from those denoted by A.

On the third view, the circle A stands for the cases in which the attribute connoted by A occurs, and the circle B for the cases in which the attribute connoted by B occurs; and the diagram shows that the two sets do not coincide, even in a single instance.

On all the three views, then, the diagram represents the meaning of an E proposition, and shows that both A and B are taken in their entire extent, or in all cases wherever they are found. This last fact is what is meant by saying that *both the subject and the predicate of an E proposition are distributed.*

§ 3. I stands for any Particular Affirmative proposition of the form 'Some A is B.' The meaning of 'some' in logical propositions, as we have already noted, is 'not none,' 'at least one.' It does not mean a *part only*. Its universal and necessary meaning is, at least one; but it does not necessarily exclude the rest. It may mean 'many,' 'most,' 'nearly the whole,' and does not exclude 'the whole' or 'all.' In accordance with this signification of the word 'some,' the proposition 'Some A is B' is represented by the following four diagrams, each of which shows that at least one A is B.

I, 1. I, 2. I, 3. I, 4.



On the first view the meaning of I is that at least one thing, and that, it may be, every thing, denoted by A, has the attribute connoted by B; and this is represented by the diagrams thus:—each of them shows that at least one thing or a part of the things coincides with the cases, while two of them (I, 3 and I, 4) show also that the whole of A may coincide with B.

On the second view the meaning of I is that at least one thing, and that, it may be, every thing denoted by A, is included

in the class denoted by B; and this is, as in the preceding case, represented by the diagrams.

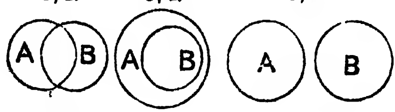
On the third view the meaning of I is that in at least one case, and that, it may be, in every case, in which the attribute connoted by A occurs, there occurs the attribute connoted by B; and this is, as in the preceding cases, represented by the diagrams.

On all the views, both the subject and the predicate are always taken in a partial extent, and sometimes also in the whole of their extent. This fact is what is meant by saying that *both the subject and the predicate of an I proposition are undistributed.*

§ 4. O stands for any Particular Negative proposition of the form 'Some A is not B.' In accordance with the logical meaning of the word 'some,' as given above, it is represented by the following three diagrams, each of which shows that at least one A is not B.

On the first view, the meaning of O is that at least one thing, and that, it may be, every thing, denoted by A, has not the

O, 1. O, 2. O, 3.



attribute connoted by 'B,'—that all the cases in which the attribute occurs are excluded from at least one thing, and, it may be, from every thing, denoted by A.

On the second view the meaning is, that at least one thing, and that it may be every thing, denoted by 'A' does not belong to the class denoted by 'B'; that the whole of the latter class is excluded from at least one, and it may be from every, individual of the former.

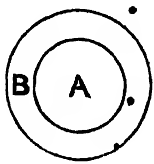
On the third view the meaning is, that in at least one case, and that it may be in every case, in which the attribute connoted by 'A' occurs, the attribute connoted by 'B' does not occur,

that every case of the latter is excluded from at least one case, and it may be from every case, of the former.

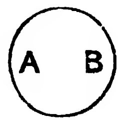
On all the views, 'B' is always taken in its entire extent, 'A' always in a part, and sometimes also in the whole of its extent. This fact is, what is meant by saying that *the predicate of an O proposition is distributed and the subject undistributed.*

§ 5. Recapitulation.—Representing 'A' and 'B,' the subject and the predicate of a proposition, by two circles, and the copula, by the mutual position or relation of the two circles, A is represented by the two diagrams (1) and (2),

(1)

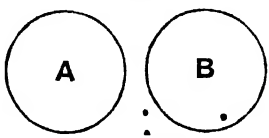


(2)



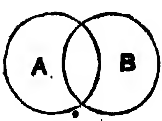
E by the single diagram (3),

(3)

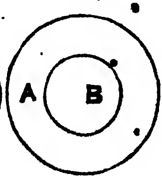


I by the four diagrams (4), (5), (6), and (7),

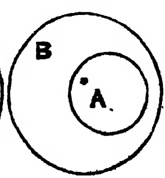
(4)



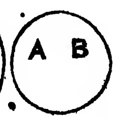
(5)



(6)

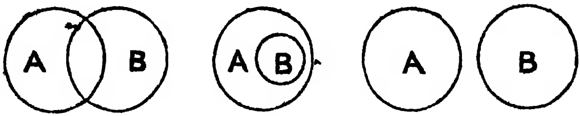


(7)



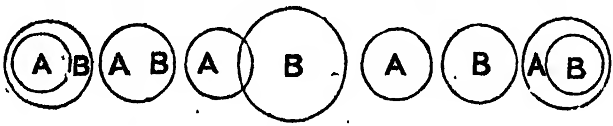
and O by the three diagrams (8), (9), and (10).

(8) (9) (10)



On a comparison of these diagrams, it will be seen that (1) and (6), (2) and (7), (3) and (10), (4) and (8), (5) and (9) are identical, and that there are altogether five fundamental diagrams. To help the memory of the student, these five diagrams are given below in a definite order:—

1st. 2nd. 3rd. 4th. 5th.



These diagrams will be henceforth called the 1st, 2nd, 3rd, 4th, and 5th respectively, and the student is advised to remember their respective numbers. A is represented by the 1st and 2nd, E by the 4th, I by the 1st, 2nd, 3rd, and 5th, and O by the 3rd, 4th, and 5th.

The subject of A is distributed, and the predicate undistributed. Both the subject and predicate of E are distributed. Both the subject and predicate of I are undistributed. The predicate of O is distributed, and the subject undistributed. That is, only universal propositions distribute their subjects, and only negative propositions distribute their predicates.

§ 6. **Exercises on the meaning and representation of propositions by diagrams.**

I. Show how the four propositional forms—*viz.*, A, E, I, and O —may be represented by diagrams.

II. Draw the five fundamental diagrams representing all propositions in their proper order, and state which of them represent A, which E, which I, and which O respectively.

III. Which of the four propositional forms—A, E, I, and O—may be represented by the 1st, which by the 2nd, which by the 3rd, which by the 4th, and which by the 5th diagram?

IV. Name the diagrams which represent A, E, I, and O respectively.

V. Represent each of the following propositions by its appropriate diagrams, and state its meaning according to the various theories of predication and of the import of propositions:

1. All men are rational.
2. All men are fallible.
3. Some men are rich.
4. Some elements are not metals.
5. Rain is produced by clouds.
6. Some plants have flowers.
7. All material bodies are extended.
8. No man is perfect.
9. All metals are elements.
10. All sensations are feelings.
11. Material bodies gravitate.
12. Silver is white.
13. Water boils at 100°C. under a pressure of 760 m.m.
14. Heat expands bodies.
15. Friction produces heat.

PART III.

REASONING OR INFERENCE.

CHAPTER I.

THE DIFFERENT KINDS OF REASONING OR INFERENCE.

(A *Reasoning* is the act of the mind by which we pass from one or more given judgments to another following from them.) When we pass from one judgment to another different from it, but contained in, or directly implied by it, the reasoning is called *Immediate.* When we pass from two or more judgments to another different from any of them, but justified by all of them jointly, the reasoning is called *Mediate.* The new judgment, or the judgment obtained from the given judgment or judgments, is called the *Conclusion,* and the given judgment or judgments, the *Premiss* or *Premisses.* If the conclusion be not more general than either of the premisses in a mediate reasoning, the reasoning is called *Deductive.* If the conclusion be, on the other hand, more general than any of the premisses, the reasoning is called *Inductive.* In Deductive Reasoning the conclusion is a development of what is contained in, or implied by, the premisses. In Inductive Reasoning the conclusion contains or implies more

than what is contained in or implied by any or all of the premisses. Thus we get the following kinds of reasoning:—

REASONING
- Immediate
- Mediate
  - Deductive
  - Inductive

Are there also two kinds, Deductive and Inductive, under Immediate Inference? Immediate Reasoning, as it is usually treated of, is all Deductive,—that is, in no case is the conclusion more general than the premiss. But if we define Immediate Reasoning as a reasoning in which a judgment is obtained from another judgment, it is evident, that the former may be more general as well as less general than the latter. If the conclusion be more general, the reasoning should certainly be called Inductive. If, for example, we could, in any case, draw the general conclusion from *a single instance*,—that is, from a single judgment or proposition—the reasoning, in that case, would be Immediate, as consisting of a single premiss only, and should be called Inductive, as leading to a conclusion more general than the premiss.

In Deductive Logic, however, all immediate reasoning and all mediate reasoning are deductive, and the following classification is, therefore, preferable:—

REASONING
- Deductive
  - Immediate
  - Mediate
    - Syllogistic
    - Non-Syllogistic: *e.g.*, certain mathematical deductive reasonings.
- Inductive

Reasoning is either Inductive or Deductive. The latter is again either (1) Immediate, or (2) Mediate, according as the conclusion follows from one premiss or from more than one. A Mediate Deductive Reasoning is called a Syllogism, when it conforms to the axiom called *Dictum de omni et nullo*,—"Whatever is affirmed or denied of a class distributively, may be affirmed or denied of any thing belonging to that class," or to some similar axiom or axioms. It may be called Mathematical, when it conforms to some one or other of the axioms in mathematics, such as (1) that things which are equal to the same thing are equal to one another, (2) that the sums of equals are equal, (3) the principle or axiom called *Argumentum a fortiori*, that 'a thing which is greater than a second, which is greater than a third, is greater than the third.' The subdivisions of the other main division cannot be discussed in this book.

A Reasoning, regarded *objectively*, is the inference of a relation from one or more given relations among things and attributes. When a general or universal relation is inferred from one, a few, or many particular relations, the reasoning or inference is Inductive. When the relation inferred is not more general than the given relation or relations, and is, in fact, contained in, or implied by, the latter, the reasoning or inference is called Deductive. It is Immediate when the inference is drawn from one given relation or premiss, and Mediate when drawn from more than one. The word inference, it should be noted, has, at least, three meanings:—(1) the process of reasoning, (2) the product of reasoning consisting of the premisses and the conclusion, and (3) the conclusion only. We have here used the word in the second sense, but it is frequently used in the first, and more frequently in the third.

A reasoning, expressed in language, is called an Argument. There are thus as many kinds or varieties of the latter as there are of the former. The simplest form of argument corresponding to the simplest form of reasoning, namely, Immediate, consists of two propositions,—the premiss and the conclusion. A Mediate deductive reasoning gives rise to an argument consisting of more than two propositions, namely, the premisses and the conclusion.

An Inductive reasoning gives rise to an argument consisting of many propositions, namely, the particular instances constituting the data, and the general conclusion justified by them. The word 'argument' also denotes a series of reasonings advanced to establish a certain conclusion.

It should be carefully noted that so far as Logic is concerned with reasoning, it treats of the principles of correct reasoning, and lays down the conditions to which reasoning must conform in order that it may be valid. It is no part of Logic to give an account of the various processes according to which men do or may reason, but of those according to which they ought to reason, and must reason if their reasonings are to be valid. The former is the business of the science of Psychology, the latter only is the business of Logic[1].

Examples of Different Kinds of Reasoning or Inference.

I. DEDUCTIVE.

i. Immediate.

1. All men are mortal,
∴ Some mortal beings are men.

2. No man is perfect,
∴ All men are imperfect.

ii. Mediate.

A.—Syllogistic.

3. All men are fallible,
All prophets are men;
∴ All prophets are fallible.

[1] No attempt is made here to give an exhaustive account of all the processes of reasoning either from the psychological or from the logical point of view. In this chapter, the subject is treated for the purposes of this work. There is great diversity of view among Logicians (1) as to the nature of reasoning or inference,—as to what is and what is not inference, and (2) as to its fundamental kinds and varieties. The theory of Reasoning and Inference, like the theory of Predication and of the Import of Propositions, is a most important subject in Logic and Psychology, and would demand a thorough treatment in a complete treatise on Logic.

4. No man is perfect,
All philosophers are men;
∴ No philosopher is perfect.

5. All metals are elements,
Gold is a metal;
∴ Gold is an element.

B.—Non-Syllogistic.

e.g., Mathematical.

6. A is equal to B,
C is equal to B;
∴ A is equal to C.

7. A is greater than B,
B is greater than C;
∴ A is greater than C.

8. A is less than B,
B is less than C;
∴ A is less than C.

9. A is a part of B,
B is a part of C;
∴ A is a part of C.

10. A is equal to B,
C is equal to D;
∴ A + C is equal to B + D.

Mathematical reasonings are usually regarded as valid, if they conform to the axioms of mathematics. By taking the axioms as major premisses, and the data of the reasonings as minor premisses, they may, however, be reduced to the syllogistic form. Examples 6 and 7 given above may be stated syllogistically as follows:—

6. Things which are equal to the same thing are equal to one another; the two things A and C are equal to the same thing (B); therefore the two things A and C are equal to one another.

7. A thing which is greater than a second, which is greater than a third, is greater than the third; the thing A is greater than a second (B), which is greater than a third (C); therefore the thing A is greater than the third (C).

Similarly, other mathematical reasonings may be reduced to fully-expressed syllogisms.

II. INDUCTIVE.

1. Air expands by heat,
Water expands by heat,
Mercury expands by heat,
Copper expands by heat,
Gold expands by heat;
∴ All material bodies expand by heat.

2. Water is solidified by cold,
Mercury is solidified by cold,
Cocoanut oil is solidified by cold;
∴ All liquids are solidified by cold.

3. The friction of the palms of our hands against each other produces heat,
The friction of two pieces of wood produces heat,
&c., &c., &c.;
∴ The friction of all material bodies produces heat.

4. Many men whom I knew have died,
All the men in the past ages have died;
∴ All men will die.

5. The three angles of this triangle are together equal to two right angles;
∴ The three angles of any triangle are together equal to two right angles.

6. These two straight lines cannot inclose a space,
∴ No two straight lines can inclose a space.

7. An equilateral triangle can be constructed upon this finite line,
∴ An equilateral triangle can be constructed upon any finite line.

Inductive reasonings conform to the canons and rules of Induction. By taking the canons and rules as major premisses, and the data of the reasonings as minor premisses, Inductive reasonings, like mathematical, may be reduced to the syllogistic form[1].

[1] See below, Appendix D.

CHAPTER II.

OF IMMEDIATE INFERENCES.

§ 1. IMMEDIATE Inference, as a process of reasoning, is the process of deriving or deducing a proposition from a given proposition or premiss. As an argument or reasoning expressed in language, it consists of the given proposition, and the proposition necessarily following from it. As an inference or conclusion, it is the proposition thus following,—the result of the process. The derivation of a proposition from a term may also be regarded as a kind of Immediate Inference. Every attribute connoted by a term may be affirmed of the term. Thus there are two kinds of Immediate Inference.

(1) In the first kind, a proposition is inferred from a term. Take the connotative term 'man,' and let its connotation consist of the two attributes 'rationality' and 'animality.' From this term it is evident that we may at once infer the following two propositions: (i) 'Man is rational,' (ii) 'Man is animal.' This kind of immediate inference depends on the axiom that every attribute connoted by a term may be predicated of it. This axiom is the basis of the formation of verbal propositions by the analysis of the connotation of terms. This mode of immediate inference is really equivalent to the affirmation of an attribute of an aggregate of attributes, or of a thing or things, of which the attribute affirmed is known to form a part.

Exercise.

Infer one verbal proposition from each of the following terms:—

(1)	Material body.	(6)	Plant.
(2)	Figure.	(7)	Animal.
(3)	Chalk.	(8)	House.
(4)	Table.	(9)	Man.
(5)	Book.	(10)	Mind.

(2) In the second kind, a proposition is inferred from a given proposition. There are seven different forms of it: *viz.*, I. Conversion; II. Æquipollence, Permutation, or Obversion; III. Contraposition; IV. Subalternation; V. Opposition; VI. Modal Consequence; VII. Change of Relation. Of these we shall treat in order.

§ 2. I.—Of Conversion.

Conversion is the admissible transposition of the subject and the predicate of a proposition. The proposition to be converted is called the *convertend*, and the proposition inferred from it the *converse*, which may be defined as a legitimate inference, having for its subject and predicate the predicate and subject, respectively, of the convertend. In an hypothetical proposition, the consequent and the antecedent are transposed. In drawing inferences by the process of conversion, the following three rules must be observed:—

(1) The subject and the predicate in the convertend must be the predicate and the subject, respectively, in the converse.

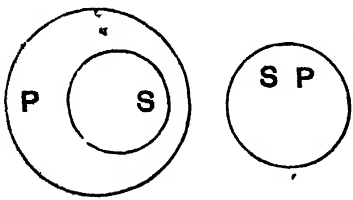
(2) No term should be distributed in the converse which was not distributed in the convertend.

(3) The quality of the converse is the same as that of the convertend,—that is, the converse of an affirmative proposition is affirmative, and the converse of a negative proposition is negative.

The first rule is evident from the definition of conversion. The second and third rules must be observed in order that the converse may be an admissible inference, that is, an inference following *necessarily* from the given proposition. The second

rule is evident from the fact that if a term is used, in the premiss, to signify *some* individuals, it can not, in the conclusion, be used to signify *every* individual, denoted by the term. The third rule follows from the meaning of an affirmative and a negative proposition. An affirmative proposition, such as S is P, means that at least one S is included in P; and from this it does not follow that at least one P is excluded from S (or P is not S), for P and S may coincide. A negative proposition, such as S is not P, means that at least one S is excluded from P; and from this it does not follow that at least one P is included in S (or P is S), for P and S may lie entirely outside of each other.

(1) From A follows I by conversion: from 'All S is P' follows by conversion 'At least one or some P is S.' This follows from the rules, and can be easily proved by the diagrams. By the third rule the converse of A must be affirmative, that is, A or I; by the second rule it can not be A; and, as no rules are violated by inferring I from A by conversion, it is I. A is represented by the first and second diagrams, and from both of these follows I, 'Some P is S.' From the first follow I, 'Some P is S,' and O, 'Some P is not S.' From the second follow A, 'All P is S,' and I, 'Some P is S.' Thus from each of them, that is, from A in every case, follows I only by conversion.



Examples.—All men are mortal: its converse is 'Some mortal is man,' 'At least one that is mortal is man,' or 'Some mortal beings are men.' If A is, B is: its converse is 'In some cases if B is, A is.' The hypothetical also can thus be converted.

(2) From I follows I by conversion: from 'Some S is P,' we can infer immediately 'At least one or some P is S.' This follows from the rules, and can be easily proved by the diagrams representing I. By the third rule the converse of I must be affirmative, that is, A or I; by the second rule it can not be A; and as

no rules are violated by inferring I from I by conversion, it is I. I is represented by the 1st, 2nd, 3rd, and 5th diagrams, and from each of them it will be seen that the converse I 'Some P is S' follows. Hence the converse of I is I[1].

Examples.—Some men are wise: its converse is 'At least one wise being is man.' In some cases if A is, B is: its converse is 'In some cases if B is, A is.'

That I follows from I by conversion and that nothing else follows may be thus shown. From the 2nd and 5th diagrams representing I, follow by conversion both A and I; from the 1st and 3rd representing I, follows by conversion I only. Thus from each of them, that is, from I in every case, follows I only by conversion.

(3) From E follows E by conversion: from 'No S is P' follows 'No P is S.' This is at once evident from the 4th diagram representing E, and follows also from the rules. By the third rule the converse of E must be negative, that is, E or O; and as no rules are violated by inferring E from E by conversion, it is E. O also follows; but it is useless to infer O where E can be inferred.

Examples.—No man is perfect: its converse is 'No perfect being is man.' If A is, B is not: its converse is 'If B is, A is not.'

(4) From O nothing follows by conversion: this follows from the rules, and can be proved by the diagrams. By the third rule the converse of O must be negative, that is, E or O; and, as the second rule is violated by inferring E or O from O by conversion, there is no converse of O.

O, 'Some S is not P,' is represented by three diagrams, *viz.*, the 3rd, 4th, and 5th.

From the 3rd follow O and I by conversion: Some P is not S, and Some P is S.

[1] The student should draw the respective diagrams in this case as well as in those that follow, and satisfy himself that the conclusions asserted to follow do really follow from them.

From the 4th follow E and O by conversion: No P is S, and Some P is not S.

From the 5th follow A and I by conversion: All P is S, and Some P is S.

Hence, from all the three forms of O, or from O in all cases, nothing follows by conversion. From the 3rd and 4th follows O; but as O does not follow from the 5th diagram, we cannot infer it from every form of O. From the 3rd and 5th follows I; but, as I does not follow from the 4th diagram, it can not be inferred from O.

Recapitulation.—The converse of I is I; and the converse of E is E. The converse in these two cases has the same quality and quantity as the convertend; and when this is the case, the process of conversion is called *Simple Conversion.* The converse of A is I. The converse, or the inferred proposition in this case, is particular, while the convertend is universal, and when this is the case, the process of conversion is called *Conversion per accidens or by limitation.* O cannot be converted.

Exercise.

Convert the following propositions:—

1. All material bodies are extended.
2. Some animals are birds.
3. No man is immortal.
4. Hydrogen is the lightest body known.
5. Benevolence is a virtue.
6. Every element is not a metal.
7. Certain metals are ductile.
8. Some animals have no power of locomotion.
9. Matter is indestructible.
10. None but elements are metals.
11. If mercury is heated, it expands.
12. If a judgment is analytical, it is not synthetical.
13. If a judgment is not synthetical, it is analytical.
14. In some cases a sensation is followed by a perception.
15. In some cases a sensation is not followed by a perception.
16. Only a man of genius can hope for success without industry.

17. All upright men are not indifferent to flattery.
18. There are few students who have a taste both for physics and metaphysics.
19. No one can hope for success without industry.
20. Knowledge is power.

§ 3. II.—Obversion, Permutation, or Æquipolence.

This process of immediate inference consists in taking the contradictory of the predicate of the given proposition as the predicate of the inference, and then changing the quality of the proposition. The inference, or the proposition inferred, is called the *Obverse* or *Permutation*, and the given proposition may be called the *Obvertend*. The obverse of a proposition may be defined as an admissible inference, having for its subject and predicate the subject and contradictory of the predicate, respectively, of the proposition.

(1) From A follows E by obversion: from the proposition 'All S is P' follows the proposition 'No S is not-P.' This is evident from the two diagrams, 1st and 2nd, representing A, from both of which follows the proposition 'No S is not-P,' 'No S is other than P.'

Example.—All men are mortal: its obverse is 'No men are not-mortal.'

(2) From E follows A by obversion: from 'No S is P' follows 'All S is not-P,' *i.e.*, every S lies in the region of not-P, or outside P. This is evident from the 4th diagram representing E.

Example.—No men are perfect: its obverse is 'All men are not-perfect.'

(3) From I follows O by obversion: from 'Some S is P' follows 'Some S is not not-P,' or 'Some S is not other than P.'

This can be proved from the diagrams: I is represented by the 1st, 2nd, 3rd, and 5th diagrams, from each of which follows the proposition 'Some S is not not-P,' *i.e.*, some S is excluded from the whole of the region belonging to not-P.

Example.—Some men are wise: its obverse is 'Some men are not not-wise.'

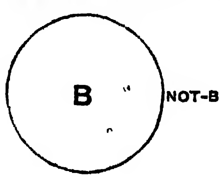
(4) From O follows I by obversion: from 'Some S is not P' follows 'Some S is not-P.' O is represented by the 3rd, 4th, and 5th diagrams, from each of which follows the proposition 'Some S is not-P,' or some S lies in the region of not-P.

Example.—Some elements are not metals: its obverse is 'Some elements are non-metals.'

An hypothetical proposition may also be obverted by taking the contradictory of the consequent as the consequent in the inference and then changing the quality of the given proposition: (1) If A is, B is: its obverse is 'If A is, not-B is not,' 'Whenever A is, nothing other than B is'[1]. (2) If A is, B is not: its

[1] With reference to this explanatory form, Mr Keynes has remarked as follows:—"Whenever A is, nothing other than B is" should hardly be given as the obverse of "If A is, B is," since 'other than' is not equivalent to 'inconsistent with,' and the existence of something other than B is compatible with B's own existence. The obverse of the given proposition should rather be stated,—"If A is, it is not true that B is not." *Mind*, for October, 1884, p. 589.

The point of Mr Keynes' objection is that 'not-B' does not mean 'other than B,' but that it means 'inconsistent with B.' I maintain that if B is taken in connotation, not-B means 'inconsistent with B,' and that if B is taken in denotation, not-B means 'other than B.' This will be evident from the following diagram:—The denotation of B is represented by the circle B, and the denotation of not-B by the region outside the circle. Here not-B includes everything except B; that is, not-B means other than B. Now, suppose the connotation of B is represented by the letter *b*, then the connotation of not-B will be any attribute inconsistent with *b*; that is, if B is taken in connotation, not-B means 'inconsistent with B,' or with the attribute connoted by B.



With this explanation of the difference in the meaning of not-B according as B is taken in denotation or in connotation, it will be seen that the first form, namely, "If A is, not-B is not," given in the text, is correct, in whatever way the terms may be interpreted—whether in denotation or in connotation—, and that the explanatory

obverse is 'If A is, not-B is,' 'Whenever A is, something other than B is.' Similarly I and O may also be obverted.

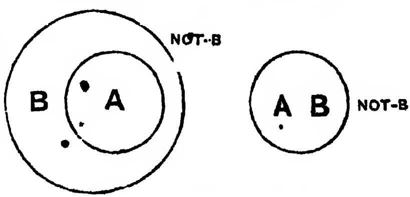
Exercise.

Obvert the following propositions:

1. All sensations are feelings.
2. Every phenomenon has a cause.
3. Only material bodies gravitate.
4. Some plants have no flowers.
5. Justice is a virtue.
6. If it rains, the ground will be wet.
7. None but elements are undecomposable.
8. If A is not B, C is D.
9. If a term is singular, it is not general.
10. If a body is heated, it rises in temperature.
11. If A is B, C is D.
12. If A is B, C is not D.
13. If A is not B, C is not D.

form, namely, "If A is, nothing other than B is," that is, "If A is anything other than B is not," is correct, if B and not-B are taken in denotation, and that Mr Keynes' form, namely, "If A is, it is not true that B is not," is correct, if B and not-B are taken in connotation.

That the forms given in the text are valid may be shown also as follows:—Reduce "If A is, B is" to the categorical form—"All A is B"—; and obvert the latter,—"No A is not-B"—; and then reduce the obverse to the hypothetical form—"If A is, not-B is not," or "If A is anything other than B is not" or "If A is nothing other than B is." This will be also evident from the two diagrams representing the given proposition.



Both shew that "If A is, not-B is not," that is, the combination A not-B does not exist.

§ 4. III.—Contraposition.

Contraposition consists in taking the contradictory of the predicate of the given proposition as the subject of the inference, and the subject as the predicate, and then changing the quality or both the quality and the quantity of the proposition, if required. The inference, or the proposition obtained by contraposition, is called the *Contrapositive.* The contrapositive of a proposition may be defined as an admissible inference, having for its subject and predicate the contradictory of the predicate and the subject, respectively, of the proposition.

(1) From A follows E by contraposition: from 'Every S is P' follows 'No not-P is S.' Here 'not-P,' the contradictory of the predicate of the given proposition (Every S is P), is taken as the subject of the inference, and the quality is changed from affirmative to negative.

This is evident from the diagrams, 1st and 2nd, representing A, from each of which follows the proposition 'No not-P is S,' *i.e.*, all S is excluded from the region of Not-P.

Example.—All men are mortal: its contrapositive is 'No not-mortal is man.'

(2) From E follows I by contraposition: from 'No S is P' follows 'Some not-P is S.' This is evident from the 4th diagram representing E. In this case the quantity of the contrapositive is particular, while the given proposition is universal.

Example.—No man is perfect: its contrapositive is 'Some not-perfect is man.'

(3) From O follows I by contraposition: from 'Some S is not P' follows 'Some not-P is S.' This may be proved from the diagrams, 3rd, 4th and 5th, representing O:—

From the 3rd follows I by contraposition: Some not-P is S. From the 4th and 5th also follows I. Hence from each of the three forms, or from O in every case, follows I by contraposition.

Example.—Some elements are not metals: its contrapositive is 'Some non-metals are elements.'

(4) From I follows no conclusion by contraposition. This may be proved thus:—

I is represented by the 1st, 2nd, 3rd, and 5th diagrams.

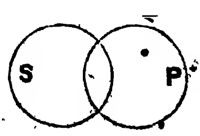
From the 3rd and also from the 5th follows by contraposition I, Some not-P is S. But from the 1st and 2nd, I does not follow. Hence from all the forms of I, that is, from I in every case, I (Some not-P is S) cannot be inferred by contraposition.

Again, from the 1st and 2nd follows O (Some not-P is not S); but it does not follow from the other two diagrams[1], and therefore O (Some not-P is not S) cannot be inferred from all the forms of I.

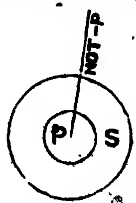
Two diagrams (3rd and 5th) allow I, and two others (1st and 2nd) allow O; but from *each* of them neither I nor O can be inferred. Hence I cannot be contraposed.

Recapitulation.—The contrapositive of A is E, of E I, and of O I, while I cannot be contraposed. The student should care-

[1] In the 3rd diagram, a part of P coincides with a part of S, and some not-P, which lies outside P and consequently outside the coinciding part of P, lies outside the coinciding part of S and not outside the whole of S,—that is, all that is known certainly is that some not-P is excluded from a part, and not from the whole, of S; or, in other words, the proposition "Some not-P is not S" is not true. In the 5th diagram, P coincides with a part of S, and therefore some not-P, which lies outside P, lies outside the coinciding part of S; but whether some not-P lies outside the remaining part of S is not known, —that is, it is not known if some not-P is excluded from the whole of S. We know only that it is excluded from a part. Hence the proposition "Some not-P is not S" is not true. This proposition means that at least one not-P is excluded from the *whole* of S; but this cannot be inferred, as we have seen, from these two diagrams.



3rd.



5th.

fully note that I cannot be contraposed, and that O cannot be converted.

An hypothetical proposition may be contraposed by taking the antecedent and the contradictory of the consequent in the proposition as the consequent and the antecedent respectively in the inference, and then changing the quality in the case of A and O, and also the quantity in the case of E.

(1) If A is, B is: its contrapositive is 'If B is not, A never is,' 'Wherever B is not, A never is.'

(2) If A is, B is not: its contrapositive is 'In some cases if B is not, A is.'

(3) In some cases if A is, B is not: its contrapositive is 'In some cases if B is not, A is.'

NOTE.—Contraposition is also called Conversion by Negation. The older logicians converted O by this process. We have seen that the process is applicable also to A and E, and inapplicable to I only. The contrapositive of a given proposition may be regarded as the *converted obverse* of it; and contraposition as consisting in obversion and in conversion of the obverse. Some logicians have indeed regarded the inference as double and the process as two-fold, including obversion and conversion, and have accordingly excluded contraposition from Immediate inference. But we have seen that, with the aid of the diagrams, the contrapositive of a proposition can be inferred as immediately as its obverse or its converse. In contraposing a proposition according to the older method, first obvert it, and then take the converse of the obverse.

Examples.

(1) All S is P.

Its obverse is 'No S is not-P'; the converse of this obverse is 'No not-P is S,' and this last is the contrapositive of the given proposition (All S is P).

(2) No S is P.

Its obverse is 'All S is not P'; the converse of this obverse is 'Some not-P is S,' which is the contrapositive of the given proposition (No S is P).

(3) Some S is not P.

Its obverse is 'Some S is not-P'; the converse of this obverse is 'Some not-P is S,' and this last is the contrapositive of the given proposition (Some S is not P).

(4) Some S is P.

Its obverse is 'Some S is not not-P,' which is O, and O cannot be converted as we have seen before (*vide* pp. 127--8).

Exercise.

Contrapose the following propositions:—

1. All animals are mortal.
2. No created being is perfect.
3. All gases can be liquefied.
4. Some plants are not devoid of the power of locomotion.
5. Some animals are insentient.
6. Some substances have no cause.
7. All bodies that have inertia have weight.
8. If mercury is heated, it expands.
9. In some cases if a body is heated, its temperature does not rise.
10. In some cases a sensation is followed by a perception.
11. If A is B, C is D.
12. If A is B, C is not D.
13. In some cases if A is B, C is not D.
14. In some cases if A is B, C is D.
15. In all cases if A is not B, C is D.
16. In all cases if A is not B, C is not D.
17. In some cases if A is not B, C is D.
18. In some cases if A is not B, C is not D.

§ 5. IV.—Of Subalternation.

This process of immediate inference consists in passing from the universal to the particular, and from the particular to the universal, with the same subject and predicate, and of the same quality. By subalternation follows :—

(1) From the truth of A, the truth of I, and from the truth of E, the truth of O; but not conversely from the latter the former. Thus, if 'All S is P' be true, 'Some S is P' will also be

true; but if the latter be true, the former will not necessarily be true.

(2) From the falsity of I, the falsity of A, and from the falsity of O, the falsity of E; but not conversely the former from the latter. If 'Some S is P' be false, then 'All S is P' must also be false; if 'Some S is not P' be false, then 'No S is P' must be false; but not conversely, that is, the falsity of the particular does not follow from the falsity of the corresponding universal. 'All S is P' may be false, and still 'Some S is P' may be true. Similarly, E may be false, and the corresponding O true.

The proof lies in the fact (1) that I or O simply repeats what is already recognized as true by A or E, and (2) that what fails even in one case can not be universally true, or what holds good even in one case can not be universally denied. The proof of the converse lies in the fact (1) that something may be true or false in some cases, in at least one case, though not universally, and (2) that what is not true or false in all cases, may yet be true or false in *some* cases, in *at least one* case. The rules of inference given above may be easily proved also from the diagrams.

§ 6. V.—Of Opposition.

In a previous chapter (*vide* p. 78) we have seen that A and O, and E and I, are called, in relation to each other, *Contradictory Opposites*, that A and E are called, in relation to each other, *Contrary Opposites*, and that I and O are called *Subcontrary Opposites*. In consequence of the opposition which exists among A, E, I, and O, having the same subject and predicate, but differing in quality, or in both quality and quantity, when any one is given as true or false, the others are necessarily either true, false, or unknown. We shall now inquire into these necessary connections among them, and lay down certain general rules of immediate inference by opposition:—

(1) Given the truth of A (All S is P). From the truth of A, as illustrated by the 1st and 2nd diagrams, it follows that E is false and also that O (Some S is not P) is false.

(2) Given the falsity of A (All S is P). From the falsity of A, as represented by one or other of the 3rd, 4th, and 5th diagrams[1], follow the truth of O (Some S is not P); and also the truth in one case (4th), and the falsity in the other cases (3rd and 5th) of E, or, in other words, the doubtfulness or uncertainty of E (No S is P).

(3) Given the truth of E (No S is P). From the 4th diagram representing E, follows at once the falsity of A, and also the falsity of I (Some S is P).

(4) Given the falsity of E (No S is P). The falsity of E is represented by one or other of the 1st, 2nd, 3rd, and 5th diagrams, from which follow the truth of I, and also the truth of A in two cases (1st and 2nd), and the falsity of A in two others (3rd and 5th), or, in other words, the doubtfulness of A.

(5) Given the truth of I (Some S is P). From the 1st, 2nd, 3rd, and 5th diagrams representing I, follow at once the falsity of E and also the truth of O (Some S is not P) in two cases (3rd and 5th), and the falsity of O in the other two (1st and 2nd), or, in other words, the doubtfulness of O.

(6) Given the falsity of I (Some S is P). This is represented by the 4th diagram, from which follows at once the truth of E (No S is P), and also the truth of O.

(7) Given the truth of O (Some S is not P). This is represented by the 3rd, 4th, and 5th diagrams, from which follows at once the falsity of A, and also the doubtfulness of I.

(8) Given the falsity of O (some S is not P). This is represented by one or other of the 1st and 2nd diagrams, from which follows at once the truth of A (All S is P), and also the truth of I.

[1] The falsity of A means that the relation between the subject and the predicate can not be represented by the 1st and 2nd diagrams, and that it must be represented by one or other of the remaining three diagrams. The falsity of E, I, or O may similarly be represented by

The results we have obtained above may be thus tabulated:—

	Given.	A All S is P.	E No S is P.	I Some S is P.	O Some S is not P.
1	A true.		False.	True by Subalternation.	False.
2	A false.		Doubtful.	Doubtful by Subalternation.	True.
3	E true.	False.		False.	True by Subalternation.
4	E false.	Doubtful.		True.	Doubtful by Subalternation.
5	I true.	Doubtful by Subalternation.	False.		Doubtful.
6	I false.	False by Subalternation.	True.		True.
7	O true.	False.	Doubtful by Subalternation.	Doubtful.	
8	O false.	True.	False by Subalternation.	True.	

A comparison of the results tabulated above leads to the following conclusions and rules of immediate inference :—

(1) The falsity of	O	follows from the truth of	A.
"	I	"	E.
"	E	"	I.
"	A	"	O.
The truth of	O	follows from the falsity of	A.
"	I	"	E.
"	E	"	I.
"	A	"	O.

That is, from the falsity of a proposition follows the truth of its contradictory opposite, and from the truth of a proposition follows the falsity of its contradictory opposite. Hence the rule :—*Of two propositions related to each other as contradictory opposites, one must be true and the other false.*

(2) From the truth of A follows the falsity of E, and from the truth of E, the falsity of A ; but not conversely. That is, from the truth of a proposition follows the falsity of its contrary opposite, but not conversely from the falsity of one the truth of the other. Hence the rule :—*Of two propositions related to each other as contrary opposites, both cannot be true ; one must be false, and both may be false.*

(3) From the falsity of I follows the truth of O, and from the falsity of O follows the truth of I, but not conversely, from the truth of the one the falsity of the other. Hence the rule :—*Of two propositions related to each other as subcontrary opposites, both cannot be false ; one must be true, and both may be true.*

These rules can also be shown to be true by a consideration of the propositions themselves and by particular examples. If the proposition 'All S is P' be true, *i.e.*, if 'P' can be affirmed of every 'S,' then it can not be denied of all 'S,' nor of any one 'S,' or, in other words, both E and O must be false. Similarly, if the proposition 'No S is P' be true, *i.e.*, if 'P' can be denied of every 'S,' then it can not be affirmed of a single 'S,' or, in other words, both I and A must be false. If the proposition

'Some S is P' be true, *i.e.*, if 'P' can be affirmed of at least one 'S,' then it can not be denied of *every* 'S,' and it may or may not be denied of *some* 'S,' or, in other words, E (No S is P) must be false, and O (Some S is P) true or false, *i.e.*, doubtful. If the proposition 'Some S is not P' be true, *i.e.*, if 'P' can be denied of at least one 'S,' then it can not be affirmed universally of 'S,' and may or may not be affirmed of some 'S,' or, in other words, A must be false and I doubtful. The other cases may also be similarly proved; and the results are the same as we have given above. We shall now give some concrete examples: If 'All metals are elements' be true, then its contrary 'No metals are elements' is evidently false; and its contradictory O 'Some metals are not elements' is also false; because, in the original proposition 'elements' is affirmed of 'all metals,' and therefore it can not be denied of some. The principle of consistency requires that what is affirmed of all members of a class, must not be denied of any of them. If 'Some elements are metals' be true, then its contradictory E 'No elements are metals' must be false, and its subcontrary O 'Some elements are not metals' may or may not be true.

Exercise.

Draw the inferences which follow by subalternation and opposition from the truth of the following propositions:—

1. All material bodies are extended.
2. The virtuous are rewarded.
3. No knowledge is useless.
4. Benevolence is a virtue.
5. Few know both physics and metaphysics.
6. Every phenomenon has a cause.
7. Some substances are uncaused.
8. Some books are not useful.
9. None but elements are metals.
10. All metals except one are solid.

§ 7. VI.—Modal Consequence.

By this process an inference is drawn from a given proposition by changing its modality:—

(1) From a necessary proposition follows the corresponding assertory, or problematic proposition, but not conversely from the latter the former: from 'S must be P' can be inferred 'S is P,' or 'S may be P'; but from 'S may be P' or 'S is P, we can not infer 'S must be P.' This is evident from the fact that from a higher degree of certainty, a lower can be inferred, but not from the latter the former.

(2) From the inadmissibility of a problematic proposition follows the inadmissibility of the corresponding assertory and necessary, from the inadmissibility of an assertory proposition follows the inadmissibility of the corresponding necessary; but not conversely from the latter the former. This is evident from the fact that where a lower degree of certainty is wanting, a higher degree can not be inferred, and that where a higher degree may be wanting, a lower degree may be established. If 'S may be P' be inadmissible, then 'S is P' and 'S must be P' must also be inadmissible. But the latter may be inadmissible, and still the former may be admissible. 'All men are wise' may be inadmissible, and still the proposition 'All men may be wise' may be admissible. 'He dies' may be inadmissible, and still 'He may die' may be admissible.

§ 8. VII.—Of Change of Relation.

This mode of immediate inference consists in inferring a proposition from a given proposition by changing the relation of the latter, that is, in inferring (1) a hypothetical from a categorical, (2) a categorical from a hypothetical, (3) hypotheticals from a disjunctive, (4) a disjunctive from hypotheticals.

(1) From the categorical 'All S is P' follows the hypothetical 'If S is, P is' (A).

From the categorical 'Some S is P' follows 'In some cases if S is, P is' (I).

From 'No S is P' follows 'In all cases if S is, P never is' (E).

From 'Some S is not P' follows 'In some cases if S is, P is not' (O).

(2) From the hypothetical 'If S is, P is' follows the cate-

gorical 'Every case of the existence of S is a case of the existence of P' (A).

From 'If A is B, C is D' follows 'Every case of A being B, is a case of C being D' (A).

From the proposition 'If S is, P is not' follows 'No case of the existence of S is a case of the existence of P.'

Similarly in the case of I and O.

(3) From the disjunctive 'A is either B or C' follows, according to Mill one or the other of the two following hypotheticals :—

(1) If A is not C, A is B.
(2) If A is not B, A is C.

According to Ueberweg, two more forms may be inferred :—

(3) If A is C, A is not B.
(4) If A is B, A is not C.

The rule of inference, according to Ueberweg, is, that the truth of one alternative implies the falsity of the other, and the falsity of the one the truth of the other. According to Mill, the rule is that the falsity of the one implies the truth of the other member, but *not conversely*, and that both the members *may be* true. According to Ueberweg, therefore, the two members of a disjunctive proposition are like two contradictory propositions, which can not both be true, the truth or the falsity of the one implying, respectively, the falsity or the truth of the other; while, according to Mill, they are like two subcontrary propositions, which may both be true, the falsity of the one implying the truth of the other.

From the disjunctive propositions, "This metal is either a conductor of heat or a conductor of electricity," "He who prefers a lower pleasure in presence of a higher is either immoral or imprudent," "Some men are either prophets or philosophers," may be inferred two hypothetical propositions, as according to Mill, while, from the disjunctive propositions, "This animal is either a vertebrate or an invertebrate," "The soul is either mortal or immortal," "Every organism is either a plant or an

animal," may be inferred four hypothetical propositions, as according to Ueberweg.

(4) From the four or the two hypotheticals may again be inferred the original disjunctive as follows:—

(*a*) The four hypotheticals are:—

(1) If A is not C, A is B.
(2) If A is not B, A is C.
(3) If A is C, A is not B.
(4) If A is B, A is not C.

From (4) if the proposition 'A is B' be true, the proposition 'A is not C' is true. Again, if the latter be true, then by the Law of Contradiction the proposition 'A is C' is false. Hence, if 'A is B' be true, 'A is C' is false. Similarly, from (3) it can be proved that if 'A is C' be true, then 'A is B' is false. Hence, of 'A is C' and 'A is B,' if one be true, the other is false. Again, if 'A is B' be false, 'A is not-B' is true by the Law of Excluded Middle (*vide* p. 17, and also Ueberweg, pp. 260–3). And if 'A is not-B' be true, then from (2) 'A is C' is true. Similarly, it can be proved that if 'A is C' be false, 'A is B' is true. Hence, of 'A is B' and 'A is C,' if one be false, the other is true. Therefore, of the two propositions 'A is B' and 'A is C,' if one be true, the other is false, and if one be false, the other is true,—that is, they are the two members of the disjunctive proposition 'Either A is B or A is C,' or 'A is either B or C,' in Ueberweg's sense.

(*b*) And from the two hypotheticals may also be inferred the original disjunctive in Mill's sense. The two hypotheticals from the disjunctive, according to Mill, are—

(1) If A is not C, A is B.
(2) If A is not B, A is C.

It has been already shown above that of the two propositions 'A is B' and 'A is C,' the falsity of the one implies the truth of the other—*i.e.*, they are the two members of the disjunctive proposition 'A is either B or C' in Mill's sense.

(*c*) Is it possible to infer immediately a disjunctive proposition from a single hypothetical? This is not possible in Ueber-

weg's sense of a disjunctive. But this is possible of a disjunctive in Mill's sense. From the hypothetical 'If A is B, A is C' follows the disjunctive 'Either A is not B or A is C.' The proof is as follows :—

(1) If A is B, A is C.

By contraposing this we get,

(2) If A is not C, A is not B.

If 'A is C' be false, 'A is not C' is true by the Law of Excluded Middle; and ∴ from (2) 'A is not B' is true. Again, if 'A is not B' be false, 'A is B' is true by the same law; and ∴ from (1) 'A is C' is true. Hence, of the two propositions 'A is C' and 'A is not B,' the falsity of the one implies the truth of the other. They are, therefore, the two members of the disjunctive proposition 'Either A is not B or A is C' in Mill's sense. Thus, a disjunctive in Mill's sense can be inferred from a single hypothetical proposition; but this is not possible in Ueberweg's sense of a disjunctive.

Exercises.

I. Distinguish the following disjunctive propositions from each other, and note the ambiguity, if any, in their meaning :—

1. The individual A is either B or C.
2. An A is either B or C.
3. Some A is either B or C.
4. Every A is either B or C.
5. Either all A is B or all A is C.

II. Infer the hypothetical propositions which follow from each of the above disjunctive propositions in Mill's and also in Ueberweg's sense of a disjunctive.

III. Draw the inferences which follow from the following propositions by change of relation :—

1. Only material bodies gravitate.
2. No plant can grow without light and heat.
3. No animal can live without oxygen.
4. A mineral is either a simple or a compound substance.

5. A material body is either solid or fluid.
6. If a proposition is not real, it is verbal.
7. Hydrogen is either a metal or a non-metal.
8. If a material body is solid, it is not fluid.
9. If mercury is heated, it rises in temperature.

IV. Infer the hypothetical propositions which follow from each of the following disjunctive propositions, and then show that the disjunctive may be re-inferred from them:—

1. Every animal is either vertebrate or invertebrate.
2. The soul is either mortal or immortal.
3. Either no S is P or some S is Q.
4. Either every A is B or some C is not D.
5. Either some A is B or some A is not C.
6. Space is either finite or infinite.
7. Every object of thought is either an idea of sensation or an idea of reflection.
8. An existence is either material or mental.
9. All knowledge is either intuitive or experimental.
10. Every mental phenomenon is either a feeling, a knowing or a willing.
11. A body is either solid, liquid, or gaseous.

V. Distinguish the following disjunctive propositions, and infer the hypothetical propositions which follow from each of them:—

1. The element hydrogen is either a metal or a non-metal.
2. An element is either a metal or a non-metal.
3. Every element is either a metal or a non-metal.
4. Element is either metallic or non-metallic.

VI. Distinguish the disjunctive propositions in each of the following groups, and infer the hypotheticals which follow from each of them:—

1.
(a) A substance is either absolute or relative.
(b) Every substance is either absolute or relative.
(c) Substance is either absolute or relative.

2.
(a) Man is either rational or irrational.
(b) Every man is either rational or irrational.
(c) This man is either rational or irrational.

3. (a) This animal is either vertebrate or invertebrate.
(b) Every animal is either vertebrate or invertebrate.
(c) An animal is either vertebrate or invertebrate.

4. (a) Substance is either knowable or unknowable.
(b) A substance is either knowable or unknowable.
(c) All substances are either knowable or unknowable.

5. (a) A body is either solid or fluid.
(b) This body is either solid or fluid.
(c) Every body is either solid or fluid.
(d) All bodies are either solid or fluid.

§ 9. Additional Forms of Immediate Inference.

Given a proposition 'A ⇁[1] B' with 'A' and 'B' as its subject and predicate respectively, the propositions immediately inferred from it will be in one or other of the following forms:—

1. 'A ⇁ not-B,' with 'A' and 'not-B' as subject and predicate.

2. 'Not-A ⇁ B,' with 'not-A' and 'B' as subject and predicate.

3. 'Not-A ⇁ not-B,' with 'not-A' and 'not-B' as subject and predicate.

4. 'B ⇁ A,' with 'B' and 'A' as subject and predicate.

5. 'Not-B ⇁ A,' with 'not-B' and 'A' as subject and predicate.

6. 'B ⇁ not-A,' with 'B' and 'not-A' as subject and predicate.

7. 'Not-B ⇁ not-A,' with 'not-B' and 'not-A' as subject and predicate.

Of these forms, the 1st is called the obverse, the 4th the converse, the 5th the contrapositive of the given proposition, and these are all that we have recognized and treated of above. But it is evident, that the other forms may also be immediately inferred from the given proposition.

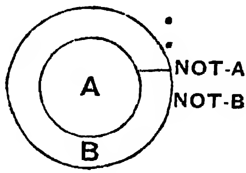
[1] This sign (⇁) is used in this place to avoid the awkward repetition of the words "is or is not."

On inspection and comparison of the diagrams of A, E, I, O, the following inferences may be easily shown to be legitimate and admissible. In proving these inferences, it is to be remembered that 'A' and 'not-A,' and 'B' and 'not-B,' cover the whole sphere of thought and existence (*vide* pp. 51—52)[1]:—

I.—From A "All A is B" follow:—

(1) No A is not-B (E, obverse).
(2) Some not-A is not B (O).
(3) Some not-A is not-B (I).
(4) Some B is A (I, converse).
(5) No not-B is A (E, contrapositive).
(6) Some B is not not-A (O).
(7) All not-B is not-A (A).

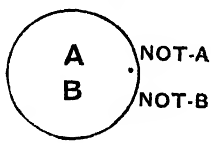
I.



II.—From E "No A is B" follow:—

(1) All A is not-B (A, obverse).
(2) Some not-A is B (I).
(3) Some not-A is not not-B (O).
(4) No B is A (E, converse).
(5) Some not-B is A (I, contrapositive).
(6) All B is not-A (A).
(7) Some not-B is not not-A (O).

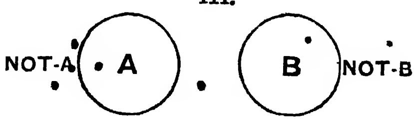
II.



III.—From I "Some A is B" follow:—

(1) Some A is not not-B (O, obverse).

III.

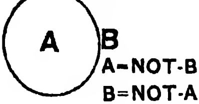


(4) Some B is A (I, converse).
(6) Some B is not not-A (O).

[1] It is of course assumed that every term, whether subject or predicate of a proposition, has a term contradictory to it.

IV.—From O "Some A is not B" follow:—

IV.



(1) Some A is not-B (I, obverse).
(5) Some not-B is A (I, contrapositive).
(7) Some not-B is not not-A (O).

The other forms in the case of I and O are wanting.

Of the seven forms given above, three—(1), (4), and (5)—have, as we have already stated, special names: obverse, converse, and contrapositive respectively; the others—(2), (3), (6), and (7)—have no special names. That these inferences are valid may be easily proved also by the older method. For example, of the inferences drawn from A, (7) is the obverse of its contrapositive, (6) is the obverse of its converse, (3) is the converse of the obverse of its contrapositive, and (2) is the obverse of the last Of the inferences drawn from E, (2) is the contrapositive of its converse, (3) is the obverse of (2), (6) is the obverse of its converse, and (7) is the obverse of its contrapositive. Thus the four additional forms may be inferred by the older method as well as by the method adopted in this work,—by the former as an inference from an inference, and by the latter as an immediate inference from the given proposition.

§ 10. Miscellaneous Exercises.

I. Give the obverse of the converse of the following propositions:—

(1) The useful is not the beautiful.
(2) Beauty is unity in variety.
(3) Wise men are few.
(4) A touches B.
(5) (*a*) I know, (*b*) I am, (*c*) He is.
(6) A is equal to B.
(7) A lies above B.
(8) The number of substances containing more than four elements is very small.
(9) Where no object is distinguished, we are not conscious of any.

(10) A is greater than B.
(11) A strikes B.
(12) A includes B.

II. Test the following inferences:—

1. Cold is agreeable;
∴ Heat is disagreeable.
2. Some elements are metals;
∴ Some non-metal is element
3. If a body is heated, it will expand;
∴ If a body expands, it is heated.
4. Some plants can move .... is true;
∴ Some plants can not move .... is also true.
5. If the rays of light fall upon the eye, they will produce the sensation of vision;
∴ If the sensation of vision is not produced, the rays of light have not fallen upon the eye.
6. All A is B.
∴ Some not-A is not-B.

III. Give the converse of the contradictory of each of the following propositions:—

1. Every man is not learned.
2. Only animals are sentient beings.
3. Nothing is annihilated.
4. If A is B, C is not D.

IV. Give the contrapositive of the contrary of each of the following propositions:—

1. Every phenomenon has a cause.
2. No man is perfect.
3. If A is B, C is D.
4. If A is B, C is not D.

V. Give the converse of the contrapositive of the contrary or sub-contrary of the contradictory of each of the following propositions:—

1. All sensations are feelings.
2. No man is immortal.
3. Some men are wise.
4. Some elements are not metals.

VI. Given the proposition 'Some men are not selfish' as true: state the propositions that can be inferred from it, (1) as true, (2) as false, and (3) as doubtful or unknown.

VII. Given the proposition 'All virtuous men are happy' as true: state the propositions that can be inferred from it, (1) as true, (2) as false, and (3) as doubtful or unknown.

VIII. Given the proposition 'Some men are unjust' as true: state the propositions that can be inferred from it, (1) as true, (2) as false, and (3) as doubtful or unknown.

IX. Given the proposition 'No man is infallible' as true: state the propositions that can be inferred from it, (1) as true, (2) as false, and (3) as doubtful or unknown.

X. Infer as many verbal or analytical propositions as you can from each of the following terms:—(1) animal, (2) matter, (3) triangle, (4) circle, (5) square, (6) man, (7) plant, (8) metal, (9) force, (10) book, (11) table, (12) horse, (13) mammal, (14) mind, (15) perception, (16) sensation, (17) house, (18) philosopher, (19) poet, (20) king, (21) nation, (22) society, (23) paper, (24) chair, (25) examination.

XI. Draw as many inferences as you can from the truth and also from the falsity of each of the following propositions:—

(1) All S is P.
(2) No S is P.
(3) Some S is P.
(4) Some S is not P.

XII. Infer as many propositions as you can from each of the following propositions being given as true:—

(1) Every phenomenon has a cause.
(2) The invariable antecedent of a phenomenon is the cause of the phenomenon.
(3) The absolute commencement of a phenomenon is not conceivable.
(4) The infinite non-commencement of a phenomenon is not conceivable.
(5) At least one substance has no cause.

CHAPTER III.

Of Syllogisms.

§ 1. A Syllogism is the inference of a proposition from two given propositions, the inferred proposition being less general than either of the two given propositions. As an argument fully expressed in language, it consists of three propositions, one of which, the conclusion, follows necessarily from the other two, called the Premisses, and thus differs from Immediate Inference, which, as the simplest and most elementary form of argument, consists of two propositions, the conclusion and the proposition from which the conclusion necessarily follows. From the proposition 'All men are mortal' follows 'Some mortal beings are men' by immediate inference,—*i. e.*, the latter is a conclusion derived from the former without the aid of any other proposition. In a Syllogism such aid is necessary, that is, a conclusion is drawn not from one proposition but from at least two propositions. For example, from the two propositions 'All men are mortal' and 'Philosophers are men,' I infer the proposition 'Philosophers are mortal.' Here (1) the conclusion follows from the two propositions taken jointly, and not from either of them singly. The two propositions must be brought together before I can legitimately infer the third which is involved in them, and yet is distinct from either. The conclusion 'Philosophers are mortal' is not the same as either of the two propositions 'All men are mortal' and 'Philosophers are men'; nor does it follow from one of them. By this character a syllogism is distinguished from an immediate inference. Again, (2) the two propositions being true, the conclusion must

be true. The one conjointly with the other makes the conclusion necessarily admissible, legitimate, or valid. By this character, a syllogism, that is, a correct or valid syllogism, is distinguished from an apparent one or a mere combination of three propositions in which the conclusion does not follow from the premisses. And (3) the conclusion can not be more general than either of the two propositions from which it is inferred. The proposition 'Philosophers are mortal' is less general than the proposition 'All men are mortal,' the latter being applicable to a much larger number of individual things than the former. By this character, a syllogism is distinguished from an induction, in which we pass from the less general to the more general, from the particular to the universal[1].

A syllogism is either pure or mixed. It is pure when both its premisses have the same relation, that is, when they are both categorical or both hypothetical; and mixed when they have different relations, that is, when one of them is hypothetical and the other categorical, or one disjunctive and the other categorical. These distinctions will be referred to more fully in a subsequent chapter[2].

§ 2. Of Categorical Syllogisms.

A Categorical Syllogism is a syllogism consisting of two categorical premisses and a categorical conclusion necessarily following from them. It is a reasoning in which a term is affirmed or denied of another by means of a third. Given two terms: if I affirm or deny one of the other, I get a categorical proposition 'A is B' or 'A is not B.' In this act there is no reasoning, mediate or immediate; there is merely an act of judgment, the direct comparison of one term with the other. If every term could be thus directly affirmed or denied of every other, there would be no such mental act as reasoning; there would be no need of it. But constituted and circumstanced as we are, we can not directly affirm or deny every term of every other. We have often to establish a relation between two terms

[1] See above, Part III, Chap. I.

[2] See below, Part III, Chap. V.

from the relation which each of them bears to a third. Given, say, two terms 'A' and 'C': it is required to find out whether A is to be affirmed or to be denied of C. Failing to do this by immediate comparison, I affirm A of every B, and B of every C, and therefrom affirm A of every C. The reasoning is thus expressed in the form of a categorical syllogism :—

Every B is A,
Every C is B;
∴ Every C is A.

In this reasoning I really compare the whole of B with A, and the whole of C with B, and thus establish a relation between the whole of C and A. I find, for example, that all the things called 'B' are included in the things called 'A,' and that all the things called 'C' are included in the things called 'B,' and conclude therefrom that all the things called 'C' are included in the things called 'A'; or I find that A-things co-exist with B-things, and that the latter co-exist with C-things, and conclude therefrom that the first co-exist with the last.

The two terms 'A' and 'C,' of which one is affirmed or denied of the other in the conclusion, are called the Extremes, while the third term B, with which each of them is compared, is called the Middle Term. The extremes occur in the premisses as well as in the conclusion, while the middle term occurs in the premisses only. The extreme, which is the subject in the conclusion, is usually called the Minor Term, and that which is the predicate, the Major Term; the premiss which contains the minor term, the Minor Premiss, and that which contains the major term, the Major Premiss.

§ 3. Whether a particular combination of three propositions constitutes a valid syllogism or not, may be easily ascertained with the aid of the diagrams used in explaining immediate inference in the preceding chapter. Draw the diagrams representing the major premiss and combine with each of them every diagram representing the minor premiss, and if the conclusion follows from each combination, then the three propositions

constitute a valid syllogism; if not, not. If the major or the minor premiss is represented by a single diagram, then combine this one with each diagram representing the other premiss, and if the conclusion follows from each combination, then the three propositions constitute a valid syllogism; if not, not. In the same way we may ascertain whether two premisses lead to any conclusion; and if so, to what conclusion. In this method of testing syllogisms, we use the following two axioms:—

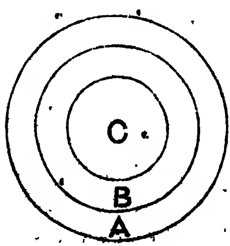
(1) Two circles coinciding with a third by any the same part coincide with each other by that part.

(2) Two circles of which one coincides and the other does not with a third by any the same part do not coincide with each other by that part.

When the first axiom is applicable, the conclusion is affirmative; when the second is applicable, the conclusion is negative; and when neither is applicable, there is no conclusion.

The truth of these axioms is evident to every person who understands the meaning of the words in which they are expressed. "Any the same part" may be "the whole" or "the smallest part possible." And the part with which one coincides may be either a part or the whole of the part with which the other coincides or does not coincide. The meaning of the words may be further illustrated by the following diagrams:—

1



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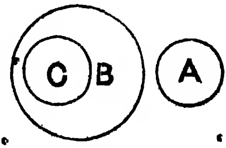

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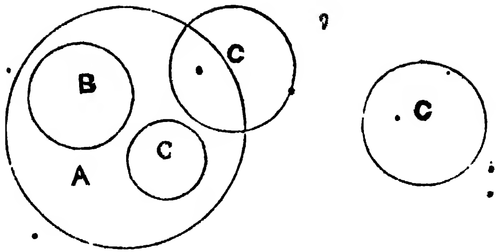


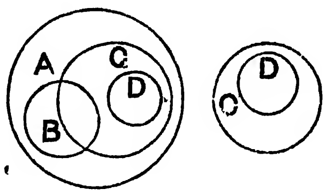
In the first diagram, two circles A and C coincide with B by any the same part,—namely, the whole of C or a part of A; therefore they coincide with each other by that part, that is, "all C is A" or "some A is C." This diagram is, in fact, a representation of the syllogism "all B is A, all C is B; therefore all C is A," and also of the syllogism "all C is B, all B is A; therefore some A is C."

In the second diagram, of the two circles C and A, C coincides with a third B by a part (the whole of C), and the other A does not coincide with B by the same part (the whole of C); therefore they do not coincide with each other by that part, that is, "no A is C," or "no C is A." This diagram is, in fact, a representation of the syllogism "all C is B, no A is B; ∴ no A is C," and also of the syllogism "no A is B, all C is B; ∴ no C is A."

In the third diagram no conclusion follows, because neither axiom is applicable to it, the circle C lying either outside or inside of the circle A.

7. § 4. By these two axioms we can distinguish a categorical syllogism, that is, a valid categorical syllogism from an apparent one, or a mere combination of three propositions in which the conclusion does not follow from the premisses. But to help the student still further in this most important process of testing syllogisms, we shall give below certain rules to which every categorical syllogism must conform. These Syllogistic Rules follow from the definition of a categorical syllogism:—

1. *Every categorical syllogism must contain three and only three terms*, neither more nor less,—namely, the two extremes between which we find a relation, and the third or middle term with which we compare each extreme in order to compare them with each other. If there be less than three, there is no means of finding the relation between the two extremes. If there be more, either there is a train of reasoning consisting of a series of syllogisms, or there is no reasoning at all. "A is B, B is C, C is D; therefore A is D." Here there are four terms, and there is a series of two syllogisms. The first two propositions give the conclusion 'A is C,' and this proposition and the next, namely, 'C is D,' allow the conclusion 'A is D.' But the following propositions containing four terms do not constitute any reasoning: "A is B, C is D, B is A, and D is C." Here there are four propositions, from which we can not infer any relation between A and C or D, or between B and C or D. This will be evident from the following figures representing the last two propositions:—



A and B may or may not lie outside C or D, that is, their relation is unknown, and can not be determined from those two propositions. It follows from this rule that no term should be ambiguous; for an ambiguous term having two distinct meanings is really equivalent to two terms, and the three terms are, in that case, really equivalent to four.

2. *Every categorical syllogism, when fully expressed, contains three and only three propositions*,—namely, the two premisses in which the middle or third term is compared with each of the two extremes, and the conclusion which expresses a relation

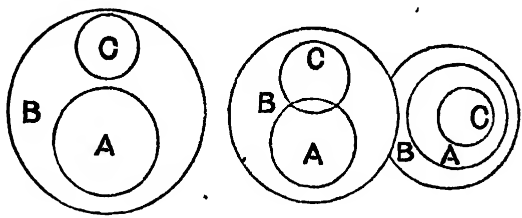
between the extremes, and which follows necessarily from the two premisses.

H 3. *The middle term must be distributed at least once.* This rule and those which are given below, follow from that part of the definition of the syllogism which requires that the conclusion must necessarily follow from the premisses. The present rule means that the middle term with which the two extremes are compared, must be taken once at least in its universal or entire extent. In other words, the whole of the circle standing for the middle term must at least once be compared with either of the two circles representing the two extremes; for otherwise one extreme might be compared with one part of the middle term, and the other with another part of it, in which case no comparison could be possible between the two extremes. This will be evident from the following diagrams:—

All A is B.

All C is B.

No conclusion.



All A and all C are each compared with a part of B, and from these two comparisons we can draw no conclusion as to the relation between C and A, that is, we can not infer that A lies outside of C, or that it lies inside of C, or that A and C intersect. This is evident from the three cases represented above. The violation of this rule leads to a fallacy, technically called the Fallacy of Undistributed Middle.

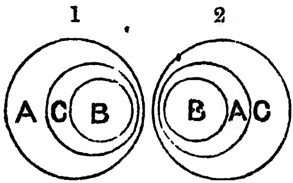
H. 4. *No term must be distributed in the conclusion which was not distributed in one of the premisses.* The non-distribution of a term in one of the premisses means that its extent has not been definitely expressed, that it has not been exactly stated whether the whole or part of its extent is meant, and that all that has been said about it is, that at least one individual or case has

been taken into consideration, while the whole is not excluded[1]. From this vagueness and indefiniteness about the extent of the term in one of the premisses, we can not, in the conclusion, take the term in its entire extent, *i.e.*, distributively. In some cases this may be allowed; but in other cases this can not be; so generally we can not distribute a term in the conclusion unless it is distributed in one of the premisses. For it must not be forgotten that what we are allowed to infer in mediate as well as in immediate inference, is not that which follows in one or two cases, but that which follows in all cases, and that if a proposition does not follow equally in all cases, it can not be regarded in Logic as a legitimate inference. This will be evident from the following diagrams:—

All B is A,

All B is C,

∴ All C is A.



From the first diagram the conclusion follows. But from the second, which also represents the premisses, it does not follow. Hence the conclusion in the general form is not true. C not being distributed in the second premiss, can not be distributed in the conclusion. The correct conclusion is 'Some C is A.'

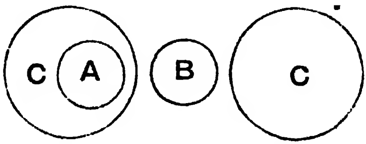
The violation of this rule leads to a fallacy, technically called the Fallacy of Illicit Process, either of the subject or of the predicate in the conclusion, that is, of the minor or of the major term.

5. *If both the premisses be negative, nothing can be inferred.* For what is expressed in the premisses is that there is no connection between the middle term and each of the two extremes; and from this nothing can be inferred between the two extremes themselves—they may or may not be connected with each other.

[1] See above, Part II, Chap. III.

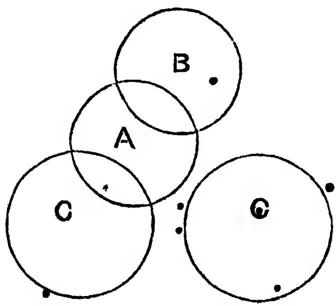
This can easily be proved by the comparison of the diagrams. A negative premiss is represented by the 3rd, 4th, and 5th diagrams.

Take the 4th and 4th. Here no conclusion follows. A and C may include each other or lie outside each other.



4th and 4th.

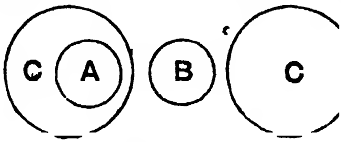
Take the 3rd and 4th. Here A and C either lie outside each other or intersect with each other, and we may infer 'Some A is not C,' but as this conclusion does not follow in the other cases, we can not infer it generally.



3rd and 4th.

Or we may prove the rule thus. The negative premisses must be either EE, EO, or OO in any order; and it will be seen, on the comparison of the diagrams, that no conclusion follows generally from any of these combinations of premisses, i.e., from each particular case of each combination. A conclusion may

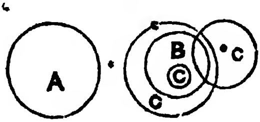
follow in *one* case of a combination, but if it does not follow in the *other* cases, it can not be regarded as a legitimate conclusion of that combination. The following diagram represents a case



4th and 4th.

namely, 4th and 4th, of each of the three combinations; and from this no conclusion follows, as we have already seen.

6. *If one premiss be negative, the conclusion must be negative.* That is, in those cases in which the conclusion does follow, it must be negative; for there may be cases in which no conclusion follows. The negative premiss merely expresses that there is no connection between the middle term and one of the extremes, and the other premiss, which must be affirmative, expresses that there is some connection between the middle term and the other extreme. From this all that we can infer is, that there is no connection between the two extremes. The negative premiss may be represented by two circles A and B lying outside each other, and the affirmative premiss by the circle B and another C, either including each other, or intersecting, or coinciding with each other. In all these different cases a part of C must be within B, which lies outside A.



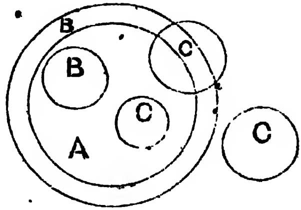
Hence we may infer that a part of C lies outside A, or "Some C is not A," a negative conclusion.

To prove the rule more satisfactorily we may have recourse to the following method. The possible premisses are AE, AO, IE, IO in any order. It will be seen from the comparison of the diagrams that in those cases in which a conclusion follows, the conclusion is negative.

Take, for example, the combination AE. It has the following different cases:—

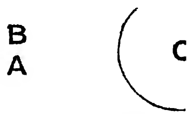
The 1st diagram and 4th, 2nd and 4th, 4th and 1st, and 4th and 2nd.

From the 1st and 4th follows a negative conclusion, namely, 'Some A is not C.'



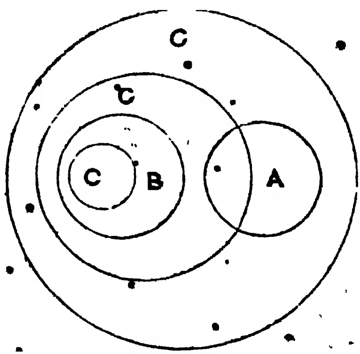
1st and 4th.

From the 2nd and 4th follows a negative conclusion, namely, 'No C is A.'



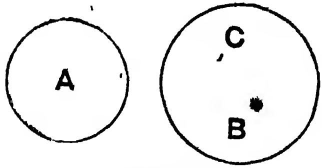
2nd and 4th.

From the 4th and 1st follows a negative conclusion, namely, 'Some C is not A.'



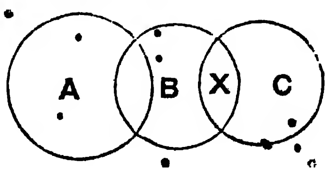
4th and 1st.

From the 4th and 2nd follows a negative conclusion, namely, 'No C is A.'



4th and 2nd.

Conversely, it can be shown that *to prove a negative conclusion one of the premisses must be negative.* A negative conclusion means that there is no connection between the two extremes, and this can only be proved by a premiss which expresses that there is no connection between the middle term and one of the extremes, and a premiss which expresses that there is a connection between the middle term and the other extreme, *i.e.*, by a negative and an affirmative premiss. A negative conclusion, for example, 'Some C is not A' means that at least a part of C lies outside the whole of A. In order to prove this, the following premisses are necessary,—1*st*, that a part of C coincides with a part of B, and 2*ndly*, that the part of B which coincides with a part of C lies outside the whole of A, the first being an affirmative and the second a negative premiss.



Here the *crossed* part of C coincides with the crossed part of B that lies outside the whole of A, therefore the crossed part of C lies outside the whole of A.

7. *If both the premisses are affirmative, the conclusion must be affirmative.* For, if the conclusion be negative, one of the premisses must be negative by the converse of Rule 6; but both

the premisses are, by supposition, affirmative; therefore the conclusion must be affirmative. Conversely, it can be shown that *to prove an affirmative conclusion, both the premisses must be affirmative.* For, if one of the premisses be negative, the conclusion will, by Rule 6, be negative; therefore both the premisses must be affirmative.

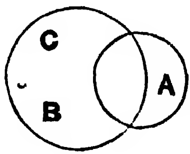
8. *If both the premisses be particular, nothing can be inferred.* The two particular premisses are either II, IO, or OO in any order. In the first combination the middle term is not distributed in either of the premisses. In the second, it may be distributed by being the predicate in O, but as the conclusion must be negative, a term will be distributed, also, in the conclusion, which was not distributed in the premisses; hence there will be an illicit process either of the subject or of the predicate in the conclusion. No conclusion follows from the last combination, both the premisses being negative. Hence it is true universally that nothing can be inferred if both the premisses be particular.

9. *If one of the premisses be particular, the conclusion must be particular.* If one premiss be particular, the other must be universal, for from two particular premisses nothing can be inferred.

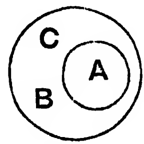
Hence, the two premisses are either IA, or IE, or OA, or OE in any order. The conclusion of IA or AI must be particular, because in the premisses only one term (the subject in A) is distributed, and that, therefore, must be the middle term; and if the conclusion were universal, a term would be distributed in it which was not distributed in the premisses; hence there would be an illicit process. The conclusion of IE or EI must be particular, for if it were universal, there would be, as in the preceding case, an illicit process. In the premisses two terms only are distributed; of these one must be the middle term, and the other one only, therefore, can be distributed in the conclusion. But the conclusion must be negative, as one of the premisses is negative, and if it were, also, universal, both its subject and predicate would be distributed; and hence there would be a

term distributed in the conclusion, which was not distributed in the premisses. Similarly, the conclusion from OA or AO must be particular; only two terms are distributed in the premisses; of these one must be the middle term, and the other the predicate of the conclusion, which will be negative, and have, therefore, the predicate distributed. Hence the subject of the conclusion must be undistributed, that is, the conclusion must be particular; otherwise there would be an illicit process. No conclusion follows from OE, as both the premisses are negative.

This rule can also be proved from the diagrams. Take the combination IA. From the 3rd and 2nd diagrams follows a



3rd and 2nd.



1st and 2nd.

particular conclusion, 'Some C is A,' and from the 1st and 2nd follows a particular conclusion, 'Some C is A.' In some cases, as in the 2nd and 2nd, a universal may follow; but as this does not follow in the other cases, it is inadmissible.

From this rule, it is evident that *if the conclusion is universal, both the premisses must be universal.* For, if one of the premisses be particular, the conclusion will be particular. Therefore both the premisses must be universal.

The last three rules, *viz.*, the 7th, 8th, and 9th, are merely consequences of the other rules. A violation of any of those three rules is a result of the violation of some of the other rules. If the other rules are carefully observed, the last three must be observed along with them, and cannot be violated.

§ 5. Division of Categorical Syllogisms into Figures.

Every valid categorical syllogism must conform to the nine rules, or conditions laid down and proved above. By the help of those rules, we can easily distinguish a valid from an invalid

categorical syllogism. Given any combination of two premisses and a conclusion, we can, by the aid of the rules, determine whether the conclusion follows from the premisses or not. When only two premisses are given, we can determine whether they lead to any conclusion, and if so, to what conclusion.

In every categorical syllogism there must be two premisses and a conclusion determined by the premisses. Given the premisses, the nature of the legitimate conclusion is given along with them. In the premisses, the middle term may have different positions in different syllogisms, and the primary division of categorical syllogisms is founded on the difference in position of the middle term in relation to the extremes in the premisses. The division is into three classes, technically called *Figures*, and is as follows:—

(1) The middle term is the subject in one premiss, and predicate in the other.

(2) The middle term is the predicate in both the premisses.

(3) The middle term is the subject in both the premisses.

Taking B to be the middle term and A and C the extremes, the three classes may be thus symbolically expressed:—

1st Class.	2nd Class.	3rd Class.
B A	A B	B A
C B	C B	B C
∴ C A or A C.	∴ C A or A C.	∴ C A or A C.

The conclusion expresses a relation between C and A, and is represented by a proposition whose subject and predicate are either A and C or C and A respectively.

If we always take C as the subject and A as the predicate in the conclusion, and call them the minor and the major term, and the two premisses in which they occur the minor and the major premiss respectively[1], we get *four classes or Figures* as follows:—

[1] It should be observed that the distinction between the major and the minor term is purely conventional. There is no reason why the subject of the conclusion should be called the minor and the predicate the major term. It is due to usage that the two names 'minor term'

1st.	2nd.	3rd.	4th.
B A	A B	B A	A B
C B	C B	B C	B C
C A	∴ C A	∴ C A	∴ C A

(1) In the 1st figure the middle term is the subject in the major premiss, and predicate in the minor premiss.

(2) In the 2nd, the middle term is the predicate in both the premisses.

(3) In the 3rd, the middle term is the subject in both the premisses.

(4) In the 4th, the middle term is the predicate in the major premiss and subject in the minor.

The conclusion is always a proposition, having C and A respectively for its subject and predicate.

The first classification or division is founded on the difference in position of the middle term in the premisses. The second is founded on this difference and on the distinction between the predicate and the subject in the conclusion, or between the major and the minor term, and the consequent distinction between the major and the minor premiss.

On the first method of classification of syllogisms there are three Figures, and on the second method there are four. On the first method the conclusion is of the form C A or of the form A C; and, on the second method, it is always of the form C A. As best adapted for teaching and as sanctioned by high authorities, we shall adopt here the four-fold classification, and take the conclusion to be always of the form C A[1].

and 'major term' are applied to the subject and the predicate, respectively, in the conclusion. The definition of the minor term is that it is the subject, and the definition of the major term is that it is the predicate, in the conclusion; in other words, the term that is the subject in the conclusion is defined as the minor term, and the term that is the predicate as the major term of a syllogism.

[1] Some logicians obtain the four figures by a double division. Ueberweg, for example, first divides all categorical syllogisms into

§ 6. Subdivision of Categorical Syllogisms in each Figure into Moods.

A syllogism may differ from another not only in the position of the middle term in the premisses, but also in the quantity and quality of the two premisses themselves. Each of the two premisses of a syllogism in each figure may consist of any one of the four propositional forms A, E, I, and O. The major premiss may be any one of these four forms, and the minor, again, may be any one of them. Thus there may be sixteen possible combinations of premisses in each figure, the first letter in each combination representing the major premiss, and the second letter the minor premiss, of a possible syllogism;—

A A	E A	I A	O A
A E	E E	I E	O E
A I	E I	I I	O I
A O	E O	I O	O O

Theoretically there can not be any other combination of premisses. All possible ones are enumerated in the list above. Of course each of these combinations does not lead to a valid conclusion, and does not, therefore, constitute a valid syllogism. By the rules given above, and by the method of the comparison of the diagrams, we shall now test these combinations, and find out which of them yield valid forms of syllogism, technically called *Moods*, and which do not, in each figure.

Of the sixteen combinations we may at once reject EE, EO, OE, and OO as invalid in all figures, because no conclusion

three chief classes, called Figures in the more comprehensive sense (the three-fold classification given above), and then subdivides the first of these three classes into two according as the middle term is the subject in the major premiss and predicate in the minor, or the predicate in the major premiss and subject in the minor, the former subdivision corresponding to the first, and the latter to the fourth of the four-fold classification given above. The second and third primary classes do not give rise to any subdivisions. The four classes thus obtained by a double division are called by him Figures in the narrower sense.

follows from two negative premisses (Rule 5). We may also reject II, IO, OI as invalid, because nothing can be inferred from two particular premisses (Rule 8).

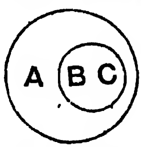
We shall now see what conclusions the remaining nine combinations AA, AE, AI, AO, EA, EI, IA, IE, and OA lead to, and which of them yield valid forms of syllogisms or moods, and which do not, in each figure.

§ 7. Valid Moods in the 1st Figure.

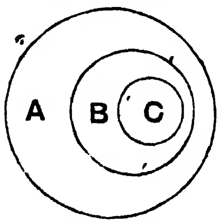
1. Take AA:—The conclusion is A. For by Rule 7, it must be affirmative, *i.e.*, A or I; and as no rule is violated by inferring A in this case, it is A.

A. All B is A,
A. All C is B;
A. ∴ All C is A[1].

That AA gives A as the conclusion in the 1st figure can be proved from the diagrams, thus:—The major premiss A is represented by the 1st and the 2nd diagram. The minor premiss A is represented by the same two diagrams. Combine each of the one with each of the other, and draw the conclusion which follows from each combination, remembering that C must be the subject, and A the predicate, in the conclusion. There are four cases, namely, the 1st and 2nd, 1st and 1st, 2nd and 1st, and 2nd and 2nd. From 1st and 2nd follows A 'All C is A.' From 1st and 1st also follows A. Similarly, from the other two cases of AA in the 1st figure follows A. AAA is, therefore, a valid mood in the first figure. From A follows I by subalternation, or I may be inferred directly from the diagrams.



1st and 2nd.



1st and 1st.

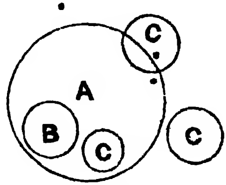
2. Take next AE:—No conclusion follows. For by Rule 6,

[1] It should be remembered that in this and in the examples that follow, B is taken as the middle term, A as the major term, C as the minor term, and CA as the typical form of the conclusion.

A. All B is A.
E. No C is B,
No conclusion.

it must be negative, *i.e.*, E 'No C is A,' or O 'Some C is not A'; but as in E and O, the major term A is distributed, while it is undistributed in the major premiss, that is, as Rule 4 is violated by inferring E or O in this case, no conclusion follows.

This can be proved from the diagrams. The major premiss A is represented by the 1st and 2nd diagrams, and the minor premiss E by the 4th diagram.



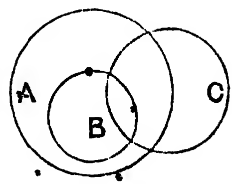
1st and 4th.

From 1st and 4th no conclusion follows, because C may be outside or inside A.

3. Take next AI:—The conclusion is I. For by Rules 7 and 9, it can not be anything else than I; and as no rule is violated by inferring I in this case, it is I.

A. All B is A,
I. Some C is B;
I. ∴ Some C is A.

This can be proved from the diagrams. The major premiss A is represented by the 1st and 2nd diagrams; and the minor premiss I by the 1st, 2nd, 3rd, and 5th diagrams. Combine each of the one with each of the other, and draw the conclusion of the form CA, which follows from each combination.



1st and 3rd.

From the 1st and 3rd follows 'Some C is A' (I). Similarly, from the 1st and 5th, 1st and 1st, 1st and 2nd, 2nd and 3rd, 2nd and 5th, 2nd and 2nd, and 2nd and 1st follows also the same conclusion.

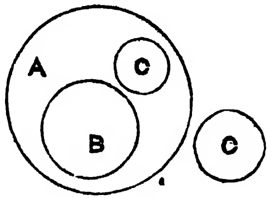
The student should draw these diagrams, and satisfy himself that the conclusion really follows from them.

A. All B is A,
O. Some C is not B,
No conclusion.

4. AO:—No conclusion follows. For by Rules 6 and 9, it can not be anything else than O; but as in O 'Some C is not A,' A is distributed, while it is undistributed in the

major premiss, that is, as Rule 4 is violated by inferring O in this case, no conclusion follows.

From the 1st and 4th diagrams, representing the major premiss A and the minor premiss O, respectively, nothing follows, because C may be outside or inside A.

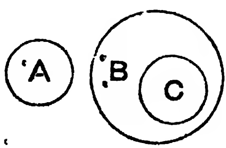


1st and 4th.

5. EA:—The conclusion is E. For by Rule 6, it must be negative, *i.e.*, E or O; and as no rule is violated by inferring E in this case, it is E.

E. No B is A,
A. All C is B;
E. ∴ No C is A.

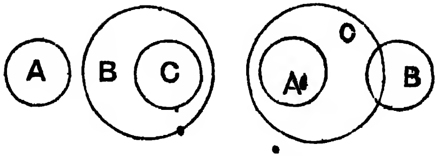
From the 4th and 1st follows E 'No C is A.' From the 4th and 2nd also follows 'No C is A.' From E follows O 'Some C is not A' by subalternation, or O may be inferred directly from the diagrams.



4th and 1st.

6. EI:—The conclusion is O. For by Rules 6 and 9, it can not be anything else than O; and as no rule is violated by inferring O in this case, it is O. From the 4th and 1st as also from the 4th and 2nd, 4th and 3rd, and 4th and 5th follows Some C is not A (O).

E. No B is A,
I. Some C is B;
O. ∴ Some C is not A.



4th and 1st. 4th and 3rd.

7. IA:—No conclusion can be drawn from this by Rule 3, because the middle term B is not distributed, being the predicate in A and the subject in I.

8. IE:—No conclusion follows. For by Rules 6 and 9, it can not be anything else than O 'Some C is not A'; but as in O, the term A is distributed in the conclusion, while it is undistributed in the major premiss, that is, as Rule 4 is violated by inferring O in this case, no conclusion follows.

9. OA:—Here the middle term is not distributed, and hence no conclusion can be drawn according to Rule 3.

In the first figure or class the combinations AA, AI, EA, and EI lead, then, to valid conclusions, and yield the following valid forms of syllogisms or *moods:* AAA, AII, EAE, EIO, technically called *Barbara, Darii, Celarent,* and *Ferio.* The conclusions of the moods AAI and EAO, which are also valid, may be inferred from the conclusions of AAA and EAE by subalternation. Hence they have been called subaltern moods, and are quite useless.

By comparing these valid moods with one another we can generalize the following two special rules of the first figure:—

(1) The major premiss must be universal. This is true of every one of the valid moods.

(2) The minor premiss must be affirmative. This is also true of every one of them.

These two special rules of the first figure may be proved thus by the general syllogistic rules. If the minor premiss be negative, the major premiss must be affirmative by Rule 5, and the conclusion negative by Rule 6, *i.e.*, A will be distributed in the conclusion, being the predicate in a negative proposition, when it

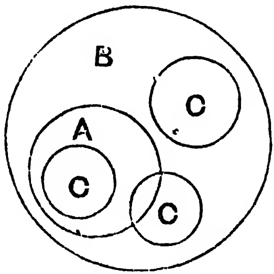
has not been distributed in the major premiss, being the predicate in an affirmative proposition. Hence the minor can not be negative; it must, therefore, be an affirmative proposition. Secondly, if the major be particular, the middle term B will not be distributed in the premisses, being the subject in a particular proposition, and predicate in an affirmative proposition. The major premiss must, therefore, be universal.

§ 8. Valid Moods in the Second Figure.

1. AA:—Nothing follows, because the middle term B is not distributed, being the predicate in two affirmative propositions.

A. All A is B,
A. All C is B,
No conclusion.

From the 1st and 1st diagrams repre-



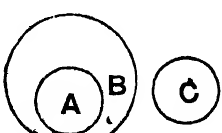
1st and 1st.

senting the major and the minor premiss A respectively, nothing follows, because C might be inside or outside A.

2. AE:—The conclusion is E. For by Rule 6, it must be negative, *i.e.*, E or O; and as no rule is violated by inferring E in this case, it is E. This can be proved from the diagrams.

A. All A is B,
E. No C is B;
E. No C is A.

The major premiss A is represented by the 1st and 2nd diagrams; and the minor premiss E by the 4th. Combine these in the usual way. From the 1st and 4th diagrams follows E 'No C is A.' From the 2nd and 4th also E follows. AEE is, therefore, a valid form



1st and 4th.

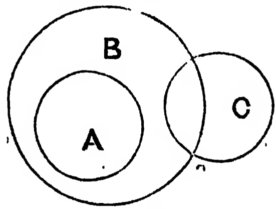
of syllogism or mood in the 2nd figure. From E follows O by subalternation, or O may be inferred directly from the diagrams.

3. AI:—Nothing follows, because the middle term is not distributed.

4. AO:—The conclusion is O. For by Rules 6 and 9, it can not be anything else than O; and as no rule is violated by inferring O in this case, it is O. The major premiss A is represented by the 1st and 2nd diagrams; and the minor premiss O by the 3rd, 4th, and 5th. Combine each of the one with each of the other.

A.	All A is B,
O.	Some C is not B;
O. ∴	Some C is not A.

From the 1st and 3rd diagrams follows O 'Some C is not A'; similarly, from the 1st and 4th, 1st and 5th, 2nd and 3rd, 2nd and 4th, 2nd and 5th also follows O. AOO is, therefore, a valid form of syllogism or mood in the 2nd figure.

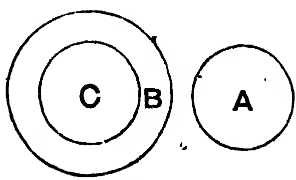


1st and 3rd.

5. EA:—The conclusion is E. For by Rule 6, it must be negative, *i.e.*, E or O; and as no rule is violated by inferring E in this case, it is E.

E.	No A is B,
A.	All C is B;
E.	No C is A.

From the 4th and 1st follows E 'No C is A,' in the 2nd figure. Similarly from the 4th and 2nd follows E. EAE is, therefore, a valid form of syllogism or mood in the second figure. From E follows O by subalternation, or O may be inferred directly from the diagrams.

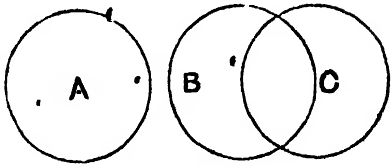


4th and 1st.

6. EI:—The conclusion is O. For by Rules 6 and 9, it can not be anything else than O; and as no rule is violated by inferring O in this case, it is O.

E.	No A is B,
I.	Some C is B;
O. ∴	Some C is not A.

From the 4th and 3rd follows O 'Some C is not A.' The part lying within B must be outside A.



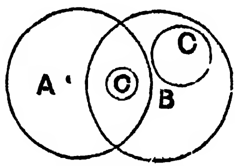
4th and 3rd.

Similarly, from the 4th and 2nd, 4th and 1st, 4th and 5th follows O 'Some C is not A.' EIO is, therefore, a valid form of syllogism or mood in the second figure.

I. Some A is B,
A. All C is B,
No conclusion.

7. IA:—Nothing can be inferred because the middle term is not distributed in the premisses.

From the 3rd and 1st nothing follows, for C may lie outside or inside A.



3rd and 1st.

8. IE:—No conclusion follows. For by Rules 6 and 9, it can not be anything else than O; but as Rule 4 is violated by inferring O in this case, no conclusion follows.

9. OA:—Nothing follows for the same reason as in the preceding case.

The valid forms of syllogism or moods in the second figure are, therefore, AEE, AOO, EAE, and EIO, technically called *Camestres*, *Baroko*, *Cesare*, and *Festino*. AEO and EAO are also valid, being merely the weakened forms of AEE and EAE; as their conclusions follow by subalternation from those of the latter, they are called subaltern moods.

From these valid moods we can generalize the following special rules of the second figure:—

(1) The major premiss must be universal.

(2) One of the two premisses must be negative.

(3) The conclusion must be negative.

Each of these rules holds good in each of the valid moods. They may be thus proved by the general syllogistic rules. If one of the premisses be not negative, the middle term will not be distributed. If one premiss be negative, the conclusion must be negative by Rule 6. The conclusion being negative, the major term, which is the predicate in it, is distributed, and must, therefore, be also distributed in the premisses; and this will not be the case, unless the major premiss be universal, because the major term is the subject in this premiss.

§ 9. Valid Moods in the Third Figure.

1. Take AA:—The conclusion is I. For by Rule 7, it must be affirmative, *i.e.*, A or I; but as Rule 4 is violated by inferring A, it can not be A; and as no rule is violated by inferring I in this case, it is I.

A. All B is A,
A. All B is C;
I. ∴ Some C is A.

∴ AAI is, therefore, a valid mood in the 3rd figure.

2. AE:—No conclusion follows. For by Rule 6, it must be negative; and as Rule 4 is violated by inferring a negative conclusion in this case, no conclusion follows.

A. All B is A,
E. No B is C,
No conclusion.

3. AI:—The conclusion is I. For by Rules 7 and 9, it can not be anything else than I; and as no rule is violated by inferring I in this case, it is I.

A. All B is A,
I. Some B is C;
I. ∴ Some C is A.

4. AO:—No conclusion follows for the same reason as in the case of AE.

A. All B is A,
O. Some B is not C,
No conclusion.

5. EA:—The conclusion is O. For by Rule 6, it must be negative, *i.e.*, E or O; but as Rule 4 is violated by inferring E, it can not be E; and as no rule is violated by inferring O in this case, it is O.

6. EI :—The conclusion is O. For by Rules 6 and 9, it can not be anything else than O; and as no rule is violated by inferring O in this case, it is O.

7. IA :—The conclusion is I. For by Rules 7 and 9, it can not be anything else than I; and as no rule is violated by inferring I in this case, it is I.

8. IE :—No conclusion follows. For by Rules 6 and 9, it can not be anything else than O; but as Rule 4 is violated by inferring O in this case, no conclusion follows.

9. OA :—The conclusion is O. For by Rules 6 and 9, it can not be anything else than O; and as no rule is violated by inferring O in this case, it is O.

That the conclusions proved above by the syllogistic rules are really valid, can be shown by the comparison of the diagrams, as in the case of the first and second figures.

The combinations AA, AI, EA, EI, IA, and OA yield, therefore, valid conclusions in the 3rd figure, and give rise to the following moods—AAI, AII, EAO, EIO, IAI, and OAO, technically called *Darapti*, *Datisi*, *Felapton*, *Ferison*, *Disamis*, and *Bokardo*.

From these valid moods we can generalize the following special rules of the third figure :—

(1) The minor premiss must be affirmative.

(2) The conclusion must be particular.

These two rules, which hold good in all the above-mentioned valid moods in the 3rd figure, may be thus proved by the general syllogistic rules. If the minor premiss be negative, the conclusion must be negative by Rule 6, and the major term, the predicate in the conclusion, will be distributed, which has not been distributed in the premisses, being the predicate in the major premiss, which must be affirmative by Rule 5. If the conclusion be universal, the minor term, the subject in the conclusion, will be distributed, which, being the predicate in the affirmative minor premiss, has not been distributed in the premisses.

§ 10. Valid Moods in the Fourth Figure.

1. AA :—The conclusion is I.	A.	All A is B,
	A.	All B is C;
	I. ∴	Some C is A.

2. AE :—The conclusion is E, and O follows from E by subalternation.

3. AI :—Invalid by Rule 3.

4. AO :—Invalid by Rule 3.

5. EA :—The valid conclusion is O.

6. EI :—The valid conclusion is O.

7. IA :—The valid conclusion is I.

8. IE :—Invalid by Rules 6 and 4.

9. OA :—Invalid by Rules 6 and 4.

In the 4th figure, the moods AAI, AEE, EAO, EIO and IAI, technically called *Bramantip*, *Camenes*, *Fesapo*, *Fresison*, and *Dimaris*, are valid. That these moods are really valid in the 4th figure can be proved by the comparison of the diagrams in the way in which we have proved the valid moods in the 1st and in the 2nd figure.

From these valid moods we can generalize the following special rules of the fourth figure:—

(1) If the major premiss be affirmative, the minor must be universal.

(2) If the minor premiss be affirmative, the conclusion must be particular.

(3) If either premiss be negative, the major premiss must be universal.

The student is required to prove these special rules by the general syllogistic rules.

§ 11. Questions and Exercises.

1. Define the following:—The major, minor, and middle terms, the major and minor premisses, the conclusion, syllogism, figure, mood, inference.

2. State the two axioms used in drawing inferences by the comparison of the diagrams.

3. Explain and illustrate the method of drawing inferences by the comparison of the diagrams.

4. Explain and illustrate the method of testing syllogisms by the comparison of the diagrams.

5. Define a syllogism, and show how the general syllogistic rules follow from its definition.

6. Prove as thoroughly as you can the following general syllogistic rules:—

(1) The middle term must be distributed at least once in the premisses.

(2) No term must be distributed in the conclusion which was not distributed in one of the premisses.

(3) If both the premisses be negative, nothing can be inferred.

(4) If one premiss be negative, the conclusion must be negative.

7. Explain fully the meaning of the terms 'figure' and 'mood.' How many figures are there? and how many moods? Give reasons for your answer.

8. Name the figure or figures in which the combination AA leads to a valid conclusion, giving reasons and concrete examples.

9. Name the figure or figures in which the combination AEE forms a valid mood, giving reasons and illustrations.

10. Give concrete examples of the following combinations of premisses in every figure, and draw the conclusions, if any, which follow from them, giving reasons:—AE, OA, IA, and IE.

11. Draw the conclusion, if any, which follows from each of the following combinations of premisses in any figure by the comparison of the diagrams:—AA, EA, AO, and EI.

12. Test by the comparison of the diagrams the following combinations or moods in every figure:—AEA, IAA, AIA, EIE, AAA, EAE.

13. Prove the following general syllogistic rules:—

(1) If both the premisses be particular, nothing can be inferred.

(2) If one of the premisses be particular, the conclusion must be particular.

(3) To prove a negative conclusion one of the premisses must be negative.

(4) If the conclusion be affirmative, both the premisses must be affirmative.

(5) If the conclusion be universal, both the premisses must be universal.

(6) If both the premisses be affirmative, the conclusion must be affirmative.

14. Prove, by the general syllogistic rules, the following special rules:—

(1) In the first figure the major premiss must be universal.

(2) In the second figure the major premiss must be universal.

(3) In the third figure the minor premiss must be affirmative.

(4) In the fourth figure one of the premisses cannot be a particular negative.

(5) In the first figure the conclusion must have the quality of the major premiss and the quantity of the minor.

(6) In the second figure the conclusion must be negative and have the quantity of the minor premiss.

(7) In the third figure the conclusion must be particular and have the quality of the major premiss.

(8) In the fourth figure the conclusion cannot be an universal affirmative.

15. Name the figure or figures (1) in which A can be proved, (2) in which E can be proved, (3) in which I can be proved, and (4) in which O can be proved.

16. Name the moods which have A, E, I, and O respectively for their conclusions.

17. Give concrete examples of the moods AII, IAI, OAO, and EAO in those figures in which they are valid.

18. State and prove the special rules of the first figure, and determine by them the valid moods in that figure.

19. State and prove the special rules of the second figure, and determine by them the valid moods in that figure.

20. State and prove the special rules of the third figure, and determine by them the valid moods in that figure.

21. State and prove the special rules of the fourth figure, and determine by them the valid moods in that figure.

CHAPTER IV.

The Aristotelian and the Scholastic Methods of determining Valid Moods.

§ 1. Aristotle's *Dictum de omni et nullo*:—This celebrated Dictum is the supreme axiom or principle of syllogistic reasoning according to Aristotle and his followers, both ancient and modern. It is thus translated by Whately: "Whatever is predicated of a term distributed, whether affirmatively or negatively, may be predicated in like manner of anything contained in it." Mill states it as follows: "Whatever can be affirmed (or denied) of a class may be affirmed (or denied) of everything included in the class." The Dictum is quite self-evident, being merely a statement of the meaning of the term *class*. A class is an indefinite number of individuals, possessing certain attributes in common. Whatever possesses those attributes belongs to the class, or is included in it. Whatever does not possess them is not included. The very condition of a thing's belonging to the class is that it must possess the attributes which are common to the individuals of the class. And the Dictum in its affirmative form simply states that what belongs to a higher class must belong to a lower, that is, to a class or to a thing included in the former, as otherwise it could not be so included; and in its negative form, it states that what does not belong to a class can not belong to any lower class or to any individual included in the former, as otherwise it could not be so included. In the proposition "All men are mortal," 'mortal' is affirmed of the class 'man,' and therefore it may be affirmed of any class, of any part of a class, or of any individual, such as 'all kings,' 'some beings,' or

'Socrates,' included in the higher class 'man.' In the proposition "No man is perfect," 'perfect' is denied of the class 'man,' and it may therefore be denied of any class, of any part of a class, or of any individual, such as 'all kings,' 'some beings,' or 'Socrates,' included in the higher class 'man.' These reasonings, stated fully, give rise to the following syllogisms:—(1) All men are mortal, all kings are men, therefore all kings are mortal; (2) all men are mortal, some beings are men, therefore some beings are mortal; (3) all men are mortal, Socrates is a man, therefore Socrates is mortal. And (1) no man is perfect, all kings are men, therefore no kings are perfect; (2) no man is perfect, some beings are men, therefore some beings are not perfect; (3) no man is perfect, Socrates is a man, therefore Socrates is not perfect.

§ 2. By applying the Dictum to the possible combinations of premisses we have given in the preceding chapter, it can be easily shown that only four (or six including the subalterns) are valid in the 1st figure, giving rise to the four moods we have already established. From the Dictum, we can easily deduce the two special rules of the 1st figure. According to the first clause of it, something must be affirmed or denied of a class distributively, that is, the major premiss must be universal, affirmative or negative. According to the last part of its second clause, something must be contained in the class, that is, the minor premiss must be affirmative. And these are the two special rules for the 1st figure. Applying the second of these two rules to the 16 combinations, we reject AE, AO, EE, OE, IE, IO, OE, and OO, and applying the first, we reject IA, II, OA, OI; and the remaining four AA, AI, EA, and EI, according to the first part of the second clause, give rise to the valid moods *Barbara*, *Darii*, *Celarent*, and *Ferio*.

§ 3. The Dictum is directly applicable to syllogisms in the 1st figure only, and can not be applied to any syllogism in the other figures. Hence Aristotle regarded the 1st figure as *perfect*, as the very type of syllogistic reasoning, and the other figures

as *imperfect.* He recognized only the first three figures, of which the first was considered to be the normal and standard mode of reasoning, and the other two as deviations from it, allowed for special purposes, as figures in Rhetoric are admissible deviations from the normal mode of expression; indeed, the word 'figure' as used in Logic has been borrowed from Rhetoric. The fourth figure is said to have been introduced by Galen, and is often called Galen's figure.

§ 4. Of Reduction:

Regarding all the figures except the first as imperfect as having no principles or axioms by which to prove syllogisms in those figures with the same cogency as the *Dictum de omni et nullo* proves those in the first, Aristotle did not recognize any syllogism as valid unless it could be transformed into one in the perfect figure, and submitted to the test of his Dictum. This transformation of a syllogism in the second, third, or fourth figure into one in the first figure is technically called *Reduction.* Whether a particular syllogism in any imperfect figure is valid or not, is to be determined by its reduction to the first. If it can be so reduced, it is valid. If not, not. Aristotle determined entirely by this method the validity of syllogistic forms in the imperfect figures. Later logicians have, by the syllogistic rules, or by the special rules, or by other methods, first determined the valid moods in those figures, and then given directions for reducing them to the first, so that the Dictum may be ultimately applied to them. Whatever method is adopted, the valid moods in the other figures are the same as those we have obtained by the joint method of the comparison of the diagrams and the syllogistic rules. The valid moods in all the figures are given in the following mnemonic verses:—

> *Barbara, Celarent, Darii, Ferioque,* prioris;
> *Cesare, Camestres, Festino, Baroko,* secundæ;
> Tertia, *Darapti, Disamis, Datisi, Felapton,*
> *Bokardo, Ferison,* habet; quarta insuper addit
> *Bramantip, Camenes, Dimaris, Fesapo, Fresison.*

These lines mean that there are four valid moods in the first figure, and four in the second, that the third figure contains six valid moods, and the fourth five. The three vowels in the name of each of the moods stand for the three propositions of the mood—the 1st for its major premiss, the 2nd for its minor premiss, and the 3rd for its conclusion. Thus the three vowels EAE in the mood *Celarent* signify that the major premiss is an E proposition, the minor an A proposition, and the conclusion an E proposition; and so with the rest.

There are two methods of reducing the imperfect moods, that is, the moods in the imperfect figures to the perfect; or rather of proving the truth of the conclusion of a mood in an imperfect figure by reduction to a perfect mood, that is, to a mood in the perfect figure:—(1) the one is called Direct or Ostensive Reduction, and (2) the other Indirect Reduction or *Reductio per deductionem ad impossibile* (*i. e.* Reduction by deduction to impossibility). In the first method the premisses of an imperfect mood are converted, obverted, contraposed, or transposed in order to form with them a mood in the first figure, having a conclusion which is the same as the original conclusion, or from which the original conclusion can be obtained by some process of immediate inference. In the second method, the truth of the conclusion of an imperfect mood is proved by showing, with the aid of the perfect moods and the rules of immediate inference by Opposition, that the contradictory of the conclusion is false.

§ 5. Ostensive Reduction:

The processes to be employed for reducing the imperfect moods by this method are indicated by certain letters contained in the names of the various moods. The initial letters B, C, D, F indicate that the imperfect moods are to be reduced to the perfect moods, having the same initial letters. The letter *s* means that the proposition signified by the vowel before it is to be converted *simply*. The letter *p* indicates that the proposition signified by the vowel before it is to be converted *by limitation* (*per accidens*). When *s* or *p* occurs after the conclusion

of an imperfect mood, *i.e.*, after the third vowel in its name, then its signification is to be applied to the conclusion of the new syllogism, that is, this conclusion must be converted simply or by limitation in order to obtain the conclusion of the imperfect mood.) The letter *m* means that the premisses of the imperfect syllogism are to be transposed. The letter *k* means that the mood containing it was reduced by the older logicians by the Indirect method. The other letters (namely *l*, *r*, *n*, *t*) are entirely meaningless, and are introduced only for phonetic purposes to make up clearly sounding words. Thus C in *Camestres* means that it is to be reduced to *Celarent; m* that the premisses are to be transposed, that is, the major premiss of this is to become the minor of the new syllogism, and the minor the major premiss; the *s* after the minor premiss, that that premiss is to be converted simply; and the *s* after the conclusion or the third vowel, that the conclusion of the new syllogism in the mood *Celarent* is to be converted simply in order to obtain the original conclusion; while the consonants *t*, *r* are entirely non-significant.

I. Take, for example, the mood *Camestres* of the 2nd figure:—

(A)	All A is B	All metals are elements,
(E)	No C is B	No compounds are elements;
(E)	∴ No C is A	∴ No compounds are metals.

By converting simply the minor premiss, and transposing the premisses of this, we get the following new syllogism in the perfect mood *Celarent*:—

(E)	No B is C	No elements are compounds,
(A)	All A is B	All metals are elements;
(E)	∴ No A is C	∴ No metals are compounds.

The converse of the conclusion of the new syllogism is the same as the conclusion of the original syllogism.

II. Take the mood *Festino* of the 2nd figure—

(E)	No A is B	No men are perfect,
(I)	Some C is B	Some beings are perfect;
(O)	∴ Some C is not A	∴ Some beings are not men.

By converting simply the major premiss we get the following:—

(E)	No B is A	No perfect beings are men,
(I)	Some C is B	Some beings are perfect;
(O)	∴ Some C is not A	∴ Some beings are not men.

This is in the perfect mood *Ferio*. Its conclusion is the same as that of the original syllogism.

III. Take the mood *Darapti* of the 3rd figure—

(A)	All B is A	All men are rational,
(A)	All B is C	All men are imperfect;
(I)	∴ Some C is A	∴ Some imperfect beings are rational.

By converting *per accidens* the minor premiss we get the following:—

(A)	All B is A	All men are rational,
(I)	Some C is B	Some imperfect beings are men;
(I)	∴ Some C is A	∴ Some imperfect beings are rational.

This is in the perfect mood *Darii*. Its conclusion is the same as that of the original syllogism.

IV. Take the mood *Felapton* of the 3rd figure—

(E)	No B is A	No men are perfect,
(A)	All B is C	All men are rational;
(O)	∴ Some C is not A	∴ Some rational beings are not perfect.

By converting *per accidens* the minor premiss we get the following:—

(E)	No B is A	No men are perfect,
(I)	Some C is B	Some rational beings are men;
(O)	∴ Some C is not A	∴ Some rational beings are not perfect.

This is in the perfect mood *Ferio*. Its conclusion is the same as that of the original syllogism.

V. Take the mood *Bramantip* of the 4th figure—

(A)	All A is B	All men are imperfect;
(A)	All B is C	All imperfect things perish;
(I)	∴ Some C is A	∴ Some perishing things are men.

By transposing the premisses we get the following :—

(A)	All B is C	All imperfect things perish,
(A)	All A is B	All men are imperfect;
(A) ∴	All A is C	∴ All men perish.

This is a syllogism in the perfect mood *Barbara*. The converse of its conclusion is the same as the conclusion of the original syllogism.

VI. Take the mood *Dimaris* of the 4th figure—

(I)	Some A is B	Some men are wise,
(A)	All B is C	All wise beings are happy;
(I) ∴	Some C is A	∴ Some happy beings are men.

By transposing the premisses we get the following:

(A)	All B is C	All wise beings are happy,
(I)	Some A is B	Some men are wise;
(I) ∴	Some A is C	∴ Some men are happy.

This is a syllogism in the perfect mood *Darii*. The converse of its conclusion is the same as the conclusion of the original syllogism.

VII. Take the mood *Fresison* of the 4th figure—

(E)	No A is B	No man is perfect,
(I)	Some B is C	Some perfect beings are infallible;
(O) ∴	Some C is not A	∴ Some infallible beings are not men.

By converting simply the major and the minor premisses we get the following :—

(E)	No B is A	No perfect being is man,
(I)	Some C is B	Some infallible beings are perfect;
(O) ∴	Some C is not A	∴ Some infallible beings are not men.

This is in the perfect mood *Ferio*. The conclusion is the same as the original conclusion.

The directions given above for reduction are not sufficient for the two imperfect moods *Baroko* and *Bokardo*. The older logicians reduced them by the method to be next described, namely, Indirect Reduction. They may be, however, reduced to the first figure, by the method of Direct Reduction, thus :—

VIII. *Baroko* of the 2nd figure—

(A)	All A is B	All men are mortal,
(O)	Some C is not B	Some beings are not mortal;
(O)	∴ Some C is not A	∴ Some beings are not men.

By contraposing the major premiss, and obverting the minor premiss, we get the following syllogism:—

(E)	No not-B is A	No immortal being is man,
(I)	Some C is not-B	Some beings are immortal;
(O)	∴ Some C is not A	∴ Some beings are not men.

This is a syllogism in the perfect mood *Ferio*, of which 'A' and 'C' are the major and minor terms, and 'not-B' the middle term.

IX. *Bokardo* of the 3rd figure—

(O)	Some B is not A	Some men are not wise,
(A)	All B is C	All men are rational;
(O)	∴ Some C is not A	∴ Some rational beings are not wise.

By contraposing the major premiss, and transposing the premisses, we get the following syllogism:—

(A)	All B is C	All men are rational,
(I)	Some not-A is B	Some not-wise are men;
(I)	∴ Some not-A is C	∴ Some not-wise are rational.

This is a syllogism in the perfect mood *Darii*, of which 'C', and 'not-A' are the major and minor terms, and 'B' the middle term. By converting simply the conclusion of the new syllogism and then obverting the converse, we can easily obtain the conclusion of the original syllogism.

The processes employed for reducing them will be sufficiently indicated if *Baroko* and *Bokardo* be called *Facoko* and *Doclamosk* respectively, *c* signifying that the proposition signified by the vowel before it is to be contraposed, *k* that the proposition is to be obverted, and *s* as usual, that the proposition is to be simply converted.

§ 6. Indirect Reduction, or, *Reductio per deductionem ad impossibile.*

I. *Baroko* of the second figure may be thus reduced by this method :—

(A) All A is B,
(O) Some C is not B;
(O) ∴ Some C is not A.

The conclusion of this syllogism is true if the premisses be true. If the conclusion 'Some C is not A' be not true, then its contradictory 'All C is A' must be true by Opposition, because of two contradictory propositions one must be true. Then combining this with the major premiss of the given syllogism, we have the following new syllogism in the perfect mood *Barbara* :—

(A) All A is B,
(A) All C is A;
(A) All C is B.

If the conclusion of this syllogism be true, its contradictory 'Some C is not B' must be false by Opposition; because of two contradictory propositions one must be false. But the latter is the minor premiss of the original syllogism, and is therefore true by supposition. Hence its contradictory, the conclusion of the new syllogism, must be false; and the falsity must be due either to the process of reasoning or to the premisses. The falsity can not be due to the process of reasoning, for the new syllogism is in the perfect mood *Barbara;* it must therefore be due to the premisses. It can not be due to the major premiss, which is also the major premiss of the original syllogism, and is therefore true by supposition: hence it must be due to the minor premiss 'All C is A,' that is, this premiss must be false, and its contradictory 'Some C is not A,' the conclusion of the original syllogism, is therefore true.

II. *Bokardo* of the 3rd figure may be thus reduced by this method :—

(O) Some B is not A,
(A) All B is C;
(O) ∴ Some C is not A.

The conclusion of this syllogism is true, if the premisses be true. If the conclusion be not true, its contradictory 'All C is A' must be true by Opposition. Then taking this as a major premiss, and the minor premiss of the original syllogism as a minor premiss, we can form the following new syllogism in the perfect mood *Barbara*:—

(A) All C is A,
(A) All B is C;
(A) ∴ All B is A.

If the conclusion 'All B is A' be true, then its contradictory 'Some B is not A' must be false by Opposition; but this is not possible, as the latter is the major premiss of the original syllogism, and therefore true by supposition. Hence the former 'All B is A' must be false; and the falsity not being due to the reasoning process which is in the perfect mood *Barbara*, nor to the minor premiss 'All B is C' of the new syllogism, which is also the minor premiss of the original syllogism, and therefore true by supposition, it must be due to the falsity of the major premiss 'All C is A.' This proposition being false, its contradictory 'Some C is not A,' the conclusion of the original syllogism, is true.

The initial letter *B* of these two moods signifies that the new syllogism which arises in the process of reduction is in the mood *Barbara*, and the letter *k* indicates that the older logicians reduced them by the Indirect method.

The Indirect method of Reduction is also applicable to the other imperfect moods.

III. Take, for example, *Cesare* of the 2nd figure—

(E) No A is B,
(A) All C is B;
(E) ∴ No C is A.

If this conclusion be not true, its contradictory 'Some C is A' must be true by Opposition. We can now form the following new syllogism in the perfect mood *Ferio*—

(E) No A is B,
(I) Some C is A;
(O) ∴ Some C is not B.

If this conclusion be true, its contradictory 'All C is B' must be false. But this is not possible, as the proposition 'All C is B' is the minor premiss of the original syllogism, and therefore true by supposition. Hence the conclusion of the new syllogism is not true; and its falsity not being due to the reasoning process, nor to the major premiss of the syllogism, must be due to the falsity of the minor premiss 'Some C is A.' Hence this proposition is false, and its contradictory 'No C is A,' the conclusion of the original syllogism, is true.

IV. Take the mood *Darapti* of the 3rd figure—

(A) All B is A,
(A) All B is C;
(I) ∴ Some C is A.

If this conclusion be not true, its contradictory 'No C is A' must be true. With this as a major premiss, and the minor premiss of the original syllogism as a minor premiss, we can form the following new syllogism in the perfect mood *Celarent*—

(E) No C is A,
(A) All B is C;
(E) ∴ No B is A.

If this conclusion be true, its contrary 'All B is A' must be false by Opposition, because two contrary propositions can not both be true, and one must be false. But 'All B is A' being the major premiss of the original syllogism can not be false; hence 'No B is A,' the conclusion of the new syllogism, can not be true and must be false, the falsity being due, as in the preceding cases, to the major premiss 'No C is A' being false. This proposition being false, its contradictory 'Some C is A,' the conclusion of the original syllogism, must be true.

§ 7. Exercises.

1. What is Reduction? Is it necessary? Define Direct and Indirect Reduction, and distinguish them from each other.

2. Reduce by the Direct method the following moods:—Cesare, Disamis, Datisi, Ferison, Bramantip, Camenes, and Fesapo.

3. Reduce the following moods by the Indirect method:—Camestres, Felapton, Bramantip, Festino, Camenes, Dimaris, and Disamis.

4. Reduce both by the Direct and by the Indirect method the two moods Baroko and Bokardo.

5. Show by the Aristotelian method that the moods AAA, EAA, AII, and AEA are invalid in the second figure.

6. Find by the same method the conclusion, if any, to which the following combinations lead in the imperfect figures:—AA, AE, EA, OA, AO, and EI.

7. Show by the same method that the moods AAA, EAE, AEE are invalid in the third figure.

8. Determine by the same method the valid moods in the second figure.

9. Give concrete examples of the following moods, and reduce them both by the Direct and by the Indirect method:—Bramantip, Disamis, Baroko, Fesapo, and Bokardo.

10. Reduce the following pairs of premisses to the first figure and draw the conclusion, if any, which follows from each pair:—

(i) No X is Y, all Y is Z.
(ii) No X is Y, all Z is Y.
(iii) All Y is X, all Y is Z.
(iv) No Y is X, all Y is Z.

11. Test the following inferences by the method of Diagrams and also by the Aristotelian and scholastic methods.

(i) No A is B; no C is not-B; therefore all C is not-A.
(ii) All A is B; all C is not-B; therefore no C is A.
(iii) No not-B is C; all not-B is A; therefore some C is not-A.
(iv) None but material bodies gravitate; air gravitates: therefore air is a material body.
(v) Plants alone have flowers; zoophytes have no flowers: therefore they are not plants.

CHAPTER V.

The Various Kinds of Syllogisms.

§ 1. A Syllogism consists of two premisses and the conclusion which follows from them. It is evident that the two premisses of a syllogism may differ in Quality, Quantity, Relation, or Modality. The various kinds or divisions of syllogisms are founded upon the modifications of these general characters of their premisses. We have seen in a previous chapter that the division into Moods is founded upon the difference in Quantity and Quality of the two premisses. The division of syllogisms into Pure and Mixed is founded upon the difference in Relation of the premisses. The division into (1) Necessary, (2) Assertory, and (3) Probable is founded upon the difference in Modality of the premisses. The various kinds or divisions may be shown thus in a tabular view :—

SYLLOGISMS.	Quality and Quantity	...	Moods.
	Relation	...	1. Pure. (Both the premisses of the same relation.) 2. Mixed. (Premisses of different relations, *e.g.*, one categorical, and the other hypothetical, &c.)
	Modality	...	1. Necessary. 2. Assertory. 3. Probable.

The two classes of Pure and Mixed syllogisms, founded on the difference in Relation of the premisses, are thus subdivided.

If the premisses of a pure syllogism are both categorical or both hypothetical, the pure syllogism is Categorical or Hypothetical. If a mixed syllogism has one premiss categorical and the other hypothetical, or one premiss categorical and the other disjunctive, or lastly, one conjunctive and the other disjunctive, it is called (1) Hypothetical-categorical, (2) Disjunctive-categorical, or (3) Conjunctive-disjunctive. By a conjunctive proposition is here meant a proposition of the form 'Neither A nor B is C' (Remotive), or of the form 'A as well as B is C' (Copulative), and it is either categorical or hypothetical. The examples we have given are categorical. The hypotheticals are of the following forms:—

If A is, neither B nor C is D (Remotive).

If A is, B as well as C is D (Copulative).

The subdivisions may be shown in a tabular view:—

SYLLOGISMS.
- I. Pure.
  1. Categorical, consisting of two categorical premisses.
  2. Hypothetical, consisting of two hypothetical premisses.
- II. Mixed.
  1. Hypothetical-categorical, consisting of one premiss hypothetical and the other categorical.
  2. Disjunctive-categorical, consisting of one premiss disjunctive and the other categorical.
  3. Conjunctive-disjunctive, consisting of one premiss conjunctive and the other disjunctive.

§ 2. I.—Of Pure Syllogisms.

The general syllogistic rules and the special rules which we have given in a previous chapter are applicable to hypothetical, as well as to categorical, syllogisms. Of the latter we have given numerous examples. We shall now give some examples of the former. In applying the general and the special rules to pure hypothetical syllogisms, we must remember (1) that the antecedent of a hypothetical proposition corresponds to the subject, and the consequent to the predicate in the corresponding categorical proposition; (2) that the quantity of a hypothetical proposition is the quantity of its antecedent, and is expressed

by such phrases as 'in all cases' and 'in some cases' or 'in one case at least,' the former denoting universal and the latter particular quantity; (3) that the quality of a hypothetical proposition is the quality of its consequent; (4) that the rules for the distribution of terms are the same as in categorical propositions, *i.e.*, the antecedent must be distributed in hypothetical propositions of the form A or E, and the consequent in hypothetical propositions of the form E or O. We shall give the following typical examples of Pure Hypothetical Syllogisms, and change them at the same time into the corresponding Categoricals :—

FIRST FIGURE.

I.—Barbara :—

A. In all cases, if B is, C is ... (major premiss),
A. In all cases, if A is, B is ... (minor premiss);
A. ∴ In all cases, if A is, C is ... (conclusion).

Changed into the corresponding categorical:

Every case of the existence of B is a case of the existence of C,
Every case of the existence of A is a case of the existence of B;
∴ Every case of the existence of A is a case of the existence of C.

II.—Celarent :—

E. In all cases, if B is, C is not . . (major premiss),
A. In all cases, if A is, B is ... (minor premiss);
E. ∴ In all cases, if A is, C is not ... (conclusion).

Changed into the corresponding categorical:

No case of the existence of B is a case of the existence of C,
Every case of the existence of A is a case of the existence of B;
∴ No case of the existence of A is a case of the existence of C.

III.—Darii :—

A. In all cases, if B is, C is ... (major premiss),
I. In some cases, if A is, B is ... (minor premiss);
I. ∴ In some cases, if A is, C is ... (conclusion).

Changed into the corresponding categorical:

Every case of the existence of B is a case of the existence of C,
Some cases of the existence of A are cases of the existence of B;
∴ Some cases of the existence of A are cases of the existence of C.

SECOND FIGURE.

IV.—Cesare:—

E. In all cases, if C is, B is not ... (major premiss),
A. In all cases, if A is, B is ... (minor premiss);
E. ∴ In all cases, if A is, C is not ... (conclusion).

Changed into the corresponding categorical:

No case of the existence of C is a case of the existence of B,
Every case of the existence of A is a case of the existence of B;
∴ No case of the existence of A is a case of the existence of C.

V.—Camestres:—

A. In all cases, if A is, B is ... (major premiss),
E. In all cases, if C is, B is not ... (minor premiss);
E. ∴ In all cases, if C is, A is not ... (conclusion).

THIRD FIGURE.

VI.—Darapti:—

A. In all cases, if B is, C is ... (major premiss),
A. In all cases, if B is, A is ... (minor premiss);
I. ∴ In some cases, if A is, C is ... (conclusion).

Similar examples may be given of the fourth figure, and also of the other moods of the first three figures.

§ 3. II.—Of Mixed Syllogisms.

We have seen that there are at least three subdivisions, namely, (1) Hypothetical-categorical, (2) Disjunctive-categorical, (3) Conjunctive-disjunctive. We shall take these in order—

1. Of Hypothetical-categorical Syllogisms.

A syllogism of this subdivision consists of a hypothetical major and a categorical minor premiss, the conclusion being categorical. The rules of inference are as follows:—

(1) If you affirm the antecedent, you may affirm the consequent of a hypothetical premiss, but not conversely, that is, it is not allowed to affirm the antecedent on affirming the conse-

quent. This rule is for what has been called a *Constructive* Hypothetical Syllogism.

(2) If you deny the consequent, you may deny the antecedent of a hypothetical premiss, but not conversely, that is, it is not allowed to deny the consequent on denying the antecedent. This rule is for what has been called a *Destructive* Hypothetical Syllogism.

Both these rules follow from the nature of the relation of dependence, expressed by a hypothetical proposition, between its antecedent and consequent. The second part of the first rule follows from the fact that the consequent may depend upon other antecedents as well as upon that antecedent, and that therefore the existence or affirmation of the consequent does not necessarily imply the affirmation of that particular antecedent, but of some one of them, and this one may not be the antecedent in question. The second part of the second rule follows from the same fact, for the consequent depending, as it may, on other antecedents as well, may exist while the particular antecedent is absent; and therefore the denial of the consequent does not follow from the denial of the antecedent. For example, in the proposition "If a person be attacked with cholera, he will die," —assuming this to be true—it does not follow that, if he be not attacked with cholera, he will not die; for he may die of consumption, fever, or some other disease Nor does it follow that if he dies, he must have been attacked with cholera, for he may die of other diseases. All that is really meant by the proposition in question is that if he gets cholera, he is sure to die; if the antecedent is present, the consequent must be present, and that if he does not die, he has not had cholera, *i.e.* if the consequent does not occur, the antecedent can not have occurred. We shall give some typical examples of Hypothetical-categorical syllogisms, and change them at the same time into the corresponding categoricals, in order to show that, when thus changed, they conform to the fundamental rules and axioms of categorical syllogisms :—

I. Constructive Hypothetical-categorical Syllogisms.

1. In all cases, if A is, B is,
   A is;
   ∴ B is.

This mode of drawing an inference is called *modus ponendo ponens*,—*i.e.* the mode which by affirming the antecedent affirms the consequent according to the first rule given above; and the syllogism has been called a *constructive* hypothetical syllogism.

It may be thus changed into a categorical:—

A. Every case of the existence of A is a case of the existence of B,
A. This is a case of the existence of A;
A. ∴ This is a case of the existence of B.

The syllogism is in the mood *Barbara*.

A Hypothetical-categorical syllogism may be also changed into a pure hypothetical syllogism; for the meaning of the minor proposition 'A is' is, that 'if this case is, A is.' By substituting this hypothetical minor premiss for the categorical, we get a pure hypothetical syllogism in the mood *Barbara*, thus:—

In all cases, if A is, B is ... ... (major premiss),
If this case is, A is ... ... (minor premiss);
∴ If this case is, B is ... ... (conclusion).

The conclusion when changed into the categorical form, is 'B is.'

The converse of the first rule does not lead to a valid syllogism—

In all cases, if A is, B is,
B is;
∴ A is.

This inference is not valid; and its invalidity can be shown by changing it into the corresponding categorical, when it will be seen that the latter violates some of the syllogistic rules, thus:—

Every case of the existence of A is a case of the existence of B,
This is a case of the existence of B.

From these two premisses no conclusion follows, because the middle term 'a case of the existence of B' is not distributed in either premiss.

2. In all cases, if A is, B is not,
A is;
∴ B is not.

This mode of drawing an inference is called *modus ponendo tollens*. Both the above *modes* (1 & 2) are called *modus ponens*, and the syllogisms in those *modes* are called *constructive* hypothetical-categorical.

It may be thus changed into a categorical:

E.	No case of the existence of A is a case of the existence of B,
A.	This is a case of the existence of A;
E. ∴	This is not a case of the existence of B.

This is a syllogism in the mood *Celarent* of the 1st figure.

It may also be changed into a pure hypothetical syllogism, thus:

E.	In all cases, if A is, B is not	(major premiss),
A.	If this case is, A is	(minor premiss);
E. ∴	If this case is, B is not	(conclusion).

Similarly, hypothetical-categorical syllogisms corresponding to *Darii* and *Ferio* may be easily formed by making the minor premiss particular.

II. Destructive Hypothetical-categorical Syllogisms.

3. In all cases, if A is, B is,
B is not;
∴ A is not.

Here the conclusion follows according to the second rule given above, and this mode of drawing an inference is called *modus tollendo tollens*,—*i.e.* the mode which by denying the consequent denies the antecedent. It may be thus changed into *Camestres* in the 2nd figure:

Every case of the existence of A is a case of the existence of B ... ... (major premiss),

This is not a case of the existence of B ... (minor premiss);
∴ This is not a case of the existence of A ... (conclusion).
In all cases, if A is, B is ... ... (major premiss),
If this case is, B is not ... ... (minor premiss);
∴ If this case is, A is not ... ... (conclusion).

The converse of the second rule does not lead to a valid syllogism. That no inference can be drawn conversely may be easily shown thus:—

In all cases, if A is, B is,
A is not;
∴ B is not.

This inference can not be drawn, as will be evident, when the syllogism is changed into the corresponding categorical:

Every case of the existence of A is a case of the existence of B,
This is not a case of the existence of A;
∴ This is not a case of the existence of B.

Here the major term 'a case of the existence of B' is distributed in the conclusion, while it is not distributed in the premiss.

4. In all cases, if A is, B is not,
B is;
∴ A is not.

Here also the conclusion is drawn according to the second rule, and this mode of inference is called *modus ponendo tollens*. Both the foregoing *modes* (3 & 4) are called *modus tollens;* and the syllogisms in those *modes* are called *Destructive* Hypothetical-categorical.

It can be easily changed into *Cesare*:—

E. No case of the existence of A is a case of the existence of B,
A. This is a case of the existence of B;
E. ∴ This is not a case of the existence of A.
In all cases, if A is, B is not ... ... (major premiss),
If this case is, B is ... ... (minor premiss);
∴ If this case is, A is not ... ... (conclusion).

To the typical forms given above may be added the following modifications of them:

5. In all cases, if A is not, B is,
A is not;
∴ B is.

It is a *constructive* hypothetical-categorical syllogism, and corresponds to the 1st example given above.

6. In all cases, if A is not, B is not,
A is not;
∴ B is not.

This is also a *constructive* hypothetical-categorical syllogism, and corresponds to the 2nd example given above.

7. In all cases, if A is not, B is,
B is not;
∴ A is.

This is a *destructive* hypothetical-categorical syllogism, and corresponds to the 3rd example given above.

8. In all cases, if A is not, B is not,
B is;
∴ A is.

This is also a *destructive* hypothetical-categorical syllogism, and corresponds to the 4th example given above. On denying the consequent, the antecedent is denied.

§ 4. 2. Of Disjunctive-categorical Syllogisms.

The next subdivision under Mixed Syllogisms is that of Disjunctive-categorical Syllogisms. In the *wider sense* a syllogism of this subdivision consists of a disjunctive and a categorical premiss, and may occur in all figures.

In the First Figure, *Barbara:*

M is either A or B ... (major premiss),
C is M ... ... (minor premiss);
∴ C is either A or B ... (conclusion).

In the Second Figure, *Camestres:*

A is either M or N ... (major premiss),

C is neither M nor N ... (minor premiss);
∴ C is not A ... ... (conclusion).

In the Third Figure, *Darapti:*

M is either A or B ... (major premiss),
M is C ... ... (minor premiss);
∴ Some C is either A or B (conclusion).

In the Fourth Figure, *Bramantip:*

A is M ... ... (major premiss),
M is either B or C ... (minor premiss);
∴ Something which is either B or C is A ... (conclusion).

In the *stricter sense* Disjunctive-categorical Syllogisms consist of the following two forms only:—

1. A is either B or C,
A is not B;
∴ A is C. *Modus tollendo ponens.*

2. A is either B or C,
A is not C;
∴ A is B. *Modus tollendo ponens.*

To these two some logicians add the following two forms:

3. A is either B or C,
A is B;
∴ A is not C. *Modus ponendo tollens.*

4. A is either B or C,
A is C;
∴ A is not B. *Modus ponendo tollens.*

Of these four forms Mill admits only the first two as valid, while Ueberweg regards all of them as equally valid. We have already referred to the difference of opinion among logicians on this subject. Mill regards the disjunctive proposition 'A is either B or C' as equivalent to one or other of the following two hypotheticals:—(1) 'If A is not B, A is C,' and (2) 'If A is not C, A is B,' and accepts accordingly the first two only of the above-mentioned four forms, while Ueberweg regards the disjunc-

tive as equivalent to one or other of the following two hypotheticals as well :—(1) 'If A is B, A is not C,' and (2) 'If A is C, A is not B,' and thus accepts all the forms.

A Disjunctive-categorical may be easily changed into a Hypothetical-categorical syllogism; and we have seen that the latter may be changed into a pure hypothetical or into a pure categorical. Thus the first may ultimately be obtained in the categorical form, and tested by the canons and rules applicable to that form, thus:—

A is either B or C ... (major premiss),
A is not B ... ... (minor premiss);
∴ A is C ... ... (conclusion).

By change of Relation we obtain from the disjunctive major the following hypothetical,—'If A is not B, A is C.' This with the other two propositions will give a hypothetical-categorical syllogism which can be easily changed into a pure one in the mood *Barbara*:

In all cases, if A is not-B, A is C, If this case is, A is not-B; ∴ If this case is, A is C.	} Hypothetical.
Every case of A being not-B, is a case of A being C, This is a case of A being not-B; ∴ This is a case of A being C.	} Categorical.

Similarly the other disjunctive-categorical forms also may be ultimately changed into the corresponding categorical forms.

§ 5. 3. Of Conjunctive-disjunctive Syllogisms, or the Dilemma.

The next and last subdivision of mixed syllogisms is the Conjunctive-disjunctive syllogism, which consists of a conjunctive and a disjunctive premiss. A conjunctive proposition has two forms—(1) Remotive, and (2) Copulative; and in each of these forms it may be categorical or hypothetical. Thus there are the following forms of it :—

1. Neither A nor B is C ... ... Remotive categorical.

2.	If A is, neither B nor C is ... ... (or Neither if A is nor if B is, is C)	Remotive hypothetical.
3.	A as well as B is C	Copulative categorical.
4.	If A is, B as well as C is ... ... (or If A is, as well as if B is, C is)	Copulative hypothetical.

The Conjunctive-disjunctive syllogism is called the Dilemma in the *wider sense*, in which the conjunctive premiss may be categorical or hypothetical, remotive or copulative, *i.e.* any one of the four forms given above, and the disjunctive premiss may be of any kind, hypothetical or otherwise. It may occur in the first as well as in the second figure.

The Conjunctive-disjunctive syllogism includes the Dilemma in the *stricter sense*, in which the conjunctive premiss is a remotive proposition, and the disjunctive premiss a hypothetical. The Dilemma in the stricter sense may be called a Hypothetical-disjunctive Syllogism, as it has, indeed, been called by some logicians. It occurs only in the second figure.

There is great difference among logicians as to the true nature and forms of the Dilemma. The view given above appears to be the best, and is taken from Ueberweg. Here I will give his definitions and forms. In the Appendix will be found the views of other logicians.

The Dilemma, Trilemma, Polylemma[1].

"In these inferences or arguments, it is shown that *whichever of the members of the disjunction may be true, the same conclusion results* (that the opponent, whichever of the different possible cases he may choose, must find himself in every case forced to the same conclusion)." They are mixed inferences or syllogisms of the 1st and especially of the 2nd figure, consisting of a Conjunctive (copulative or remotive) and a Disjunctive premiss.

"The Dilemma, in the *stricter and special sense*, is an inference of the *second figure*, with a *hypothetico-disjunctive premiss* (which is sometimes major and sometimes minor premiss), and with a *remotive premiss*."

[1] Ueberweg's *Logic*, pp. 455—57.

"In the *wider sense* of the term, inferences with a categorico-disjunctive premiss, and inferences in the first figure with a disjunctive and a copulative or remotive premiss, are also attributed to it. The like holds good of the Trilemma, Tetralemma, and Polylemma."

FORMS OF THE DILEMMA IN THE *stricter sense.*

Second Figure.

(1) If A is, either B or C is ... ... (hypothetical-disjunctive),
Neither B nor C is ... ... (remotive premiss);
∴ A is not.

(2) If A is, neither B nor C is ... (hypothetical-remotive),
If D is, either B or C is ... ... (hypothetical-disjunctive),
∴ If D is, A is not.

(3) If A is, either B or C is ... ... (hypothetical-disjunctive),
If D is, neither B nor C is ... (hypothetical-remotive);
∴ If D is, A is not.

The 1st may be thus analysed:

The major premiss, If A is, either B or C is, is equivalent to—

(1) If A is, B is,
or (2) If A is, C is;

and the remotive minor is equivalent to—

(1) B is not,
and (2) C is not.

Take the first alternative of the major premiss and the first of the minor:

If A is, B is,
B is not;
∴ A is not. *Modus tollendo tollens.*

Take the second alternative of the major premiss and the second of the minor:

If A is, C is,
C is not;
∴ A is not. *Modus tollendo tollens.*

Thus in either case, that is, whichever of the two alternatives be true, the conclusion is the same (A is not), as required by the definition.

The second may be thus analysed:

The major premiss, 'If A is, neither B nor C is,' is equivalent to—

(1) If A is, B is not,
and (2) If A is, C is not.

The minor premiss is equivalent to—

(1) If D is, B is,
or (2) If D is, C is.

Take (1) of the major and (1) of the minor—

E.	If A is, B is not	(major premiss),
A.	If D is, B is	(minor premiss);
E. ∴	If D is, A is not	(conclusion).

This is a pure hypothetical syllogism in the mood *Cesare.*

Take (2) of both the premisses—

If A is, C is not	(major premiss),
If D is, C is	(minor premiss);
∴ If D is, A is not	(conclusion).

This is also in the same mood. The conclusion is the same as required by the definition.

The third may be thus analysed:

The major premiss is equivalent to either

(1) If A is, B is,
or (2) If A is, C is;

and the minor to—

(1) If D is, B is not,
and (2) If D is, C is not.

Taking (1) of both the premisses—

A.	If A is, B is	(major premiss),
E.	If D is, B is not	(minor premiss);
E. ∴	If D is, A is not	(conclusion).

This is in the mood *Camestres.*

Taking (2) of both the premisses—

If A is, C is	(major premiss),
If D is, C is not	(minor premiss);
∴ If D is, A is not	(conclusion).

This is also in the same mood. The conclusion is the same in either case, that is, whichever member of the disjunction is accepted, the same conclusion is arrived at.

FORMS OF THE DILEMMA IN THE *wider sense.*

Second Figure.

1. A is either B or C ... ... (Categorical-disjunctive),
D is neither B nor C ... ... (Remotive);
∴ D is not A.

2. A is neither B nor C ... ... (Remotive),
D is either B or C ... ... (Categorical-disjunctive);
∴ D is not A.

3. If A is, neither B nor C is ... (Hypothetical-remotive),
Either B or C is ... ... (Categorical-disjunctive);
∴ A is not.

First Figure.

1. A as well as B is C ... ... (Copulative),
D is either A or B ... ... (Disjunctive);
∴ D is C.

2. If A is, as well as if B is, C is (Hypothetical-copulative),
If D is, either A or B is ... (Hypothetical-disjunctive);
∴ If D is, C is.

3. Neither A nor B is C ... ... (Remotive),
D is either A or B ... ... (Categorical-disjunctive);
∴ D is not C.

4. Neither if A is nor if B is, is C (Hypothetical-remotive),
If D is, either A or B is ... (Hypothetical-disjunctive);
∴ If D is, C is not.

5. Neither if A is nor if B is, is C (Hypothetical-remotive),
Now either A or B is ... ... (Disjunctive);
∴ C is not.

The first form in the first figure may be thus analysed:—

The major premiss is equivalent to—

(1) A is C,
and (2) B is C;

and the minor to either—

(1) D is A,
or (2) D is B.

From (1) of both the premisses—

A is C,
D is A;
∴ D is C.

From (2) of both—

B is C,
D is B;
∴ D is C.

The conclusion is in either case the same, 'D is C.'

The second form in the first figure may be thus analysed:—

From the major we get—

(1) If A is, C is,
and (2) If B is, C is;

and from the minor we get—

(1) If D is, A is,
or (2) If D is, B is.

From (1) of both the premisses—

If A is, C is,
If D is, A is;
∴ If D is, C is.

This is in the mood *Barbara*.

Similarly from (2) of both, we get a pure hypothetical syllogism in the same mood and with the same conclusion.

§ 6. Exercises.

Test the following arguments:—

(1) If the sun shines, it will be a brilliant day; if it is not foggy or cloudy, the sun will shine; therefore, if it is not foggy or cloudy, it will be a brilliant day.

(2) If the temperature rises, the barometer will fall; if the barometer falls, the weather will not be fine; therefore, if the temperature rises, the weather will not be fine.

(3) If a gas is subjected to a higher pressure, its volume diminishes; if its volume diminishes, its density increases; therefore, if a gas is subjected to a higher pressure, its density increases.

(4) If the earth did not rotate, there would be no alternation of day and night; there is alternation of day and night; therefore the earth does rotate.

(5) Without light and heat, no plants could grow; without plants no animals could live; man, being an animal, could not, therefore, live without light and heat.

(6) An organized being is either an animal or a plant: this substance is neither; therefore it is not an organized being.

(7) If a substance has inertia, it has gravity; if it does not resist, it has no inertia; therefore, if a substance does not resist, it has no gravity.

(8) If a substance gravitates, it has inertia; if a substance has the power of resistance, it has inertia; therefore if a substance gravitates, it has the power of resistance.

(9) If a solid is heated, it becomes a liquid; if a liquid is heated, it becomes a gas: therefore if a solid is heated, it becomes a gas.

(10) If A is not, B is not; if B is not, C is not: therefore if A is not C is not.

(11) An igneous rock is either volcanic or plutonic; trap is a kind of igneous rock: therefore it is either volcanic or plutonic.

(12) A material body is either organic or inorganic; a crystal is not organic: therefore it is inorganic.

(13) If water is heated, either its bulk increases, or its temperature rises, or it passes into vapour; neither of these changes is happening to the water in this flask: therefore it is not heated.

(14) All existences are either mental or material; nothing is neither mental nor material: therefore nothing is not an existence.

(15) A liquid as well as a gas is expanded by heat; a fluid is either a gas or a liquid: therefore a fluid is expanded by heat.

(16) If the motion of a body is impeded, heat is produced; if heat is produced, the body will either rise in temperature or increase in bulk, or pass into a different state; therefore, if the motion of a body is impeded, the body will either rise in temperature, or increase in bulk, or pass into a different state.

(17) If every notion is derived from sensation or reflection, the notion of extension is also so derived. But it cannot be so derived. Therefore every notion is not derived from sensation or reflection.—*Reid's Inquiry.*

(18) If Nature had given us nothing more than sensations corresponding to the impressions made by the objects upon the body, we should not in that case have been percipient beings. But we are percipient beings; therefore Nature has given us more than the sensations.—*Reid.*

(19) Body and spirit, cause and effect, time and space, to which we were wont to ascribe an existence independent of our thought, are all turned out of existence by this short dilemma. Either these things are ideas of sensation or reflection or they are not: if they are ideas of sensation or reflection, they can have no existence but when we are conscious of them; if they are not ideas of sensation or reflection, they are words without any meaning.—*Reid.*

§ 7. Of Enthymemes[1].

An Enthymeme is an abridged syllogism, that is, a syllogism, one of whose premisses is not expressed in language. For example, 'gold is an element, because it is a metal'; here we have a syllogism apparently consisting of two propositions, but really of three, including the major premiss, which is suppressed, and which must be as follows:—'all metals are elements.' Without this, the conclusion 'gold is an element' can not be drawn from the single premiss 'gold is a metal.' Though the major premiss is not expressed in language, it is contended that it must have been present in thought to complete or constitute the act of reasoning. Similarly, the minor premiss may be sometimes suppressed. For example, 'all men are fallible, and therefore kings or philosophers are fallible.' Here the minor premiss 'kings or philosophers are men' is understood. Sometimes even the conclusion may be suppressed, and hinted at by the expression of the two premisses, or, in rare cases, of one only. This often happens in conversation between educated persons on delicate subjects. For example, happening to talk about

[1] The word *enthymeme* originally meant a syllogism with probable premisses. This is the sense in which Aristotle used it. It came afterwards to mean a syllogism which was imperfect not from its premisses being probable, but from one of them being suppressed, and in this sense the word is now used in Logic.

a particular institution which has been much praised and declared as *perfect*, an opponent might, in reply, simply say that 'everything human is imperfect,' or that 'everything is liable to change and decay': here nothing but the major premiss is expressed, and it is of course implied that 'the institution in question is human' (minor premiss), and that 'it is, therefore, not perfect' (conclusion).

§ 8. Exercises.

1. *To supply the suppressed premiss of an Enthymeme.* (1) Note the subject and the predicate in the conclusion which are the minor and the major term, respectively, of the syllogism, and then see whether the premiss to be supplied is the major or the minor premiss. (2) If it be the major premiss, form such a proposition with the major and the middle term as will make the conclusion valid. (3) If it be the minor premiss, form such a proposition with the minor and the middle term as will make the conclusion valid. Examples:—(1) "All metals are elements, because they can not be decomposed." In this the subject and the predicate in the conclusion are respectively 'all metals' and 'elements,' and these two are, therefore, the minor and the major term, respectively. The given premiss contains the minor term 'metals,' and is, therefore, the minor premiss. The premiss suppressed is, therefore, the major premiss, and is the proposition 'all substances that can not be decomposed are elements.' (2) "Small-pox has a cause, because every phenomenon has a cause." Here 'small-pox' is the minor term, 'has a cause' the major term, and 'phenomenon' the middle term. The premiss expressed containing the major term 'has a cause,' is the major premiss. The premiss suppressed is, therefore, the minor premiss, and is the proposition 'small-pox is a phenomenon.'

2. *To find premisses for a given conclusion.* In finding premisses for a given conclusion, note the subject and the predicate in the conclusion, which must be the minor and the major term, respectively, of the required syllogism. If the conclusion be negative, find such a middle term as will form with the predicate an E proposition, and with the subject an A or I proposition. If the conclusion be affirmative, find such a middle term as will form with the predicate an A proposition, and with the subject an A or I proposition. The three

terms are to have the same relative position as in the first figure. Examples: (1) Find premisses for the conclusion 'no prophet is infallible'; here the term 'man' will do as a middle term; and the required premisses are 'no man is infallible' and 'all prophets are men.' (2) Find premisses for the conclusion 'some elements are metals'; here the term 'undecomposable substances conducting heat and electricity' will do as a middle term; and the premisses required are 'all undecomposable substances conducting heat and electricity are metals,' and 'some elements are undecomposable substances conducting heat and electricity.'

3. *To draw the conclusion, if any, which follows from two given propositions as premisses*:—See if the two premisses are in any particular valid mood in any of the four figures. If so, draw the conclusion which follows from them in accordance with that mood. If not, try to reduce them to a valid mood by verbal changes and by processes of immediate inference. If they can be thus transformed into a valid mood, draw the inference justified by that mood. If they cannot be so transformed, no conclusion follows from the two given propositions. It should be remembered that the conclusion not being given, it is not known which term is major and which minor, that the premiss stated first is not necessarily the major premiss, and the premiss stated second the minor premiss, that the two premisses may be given and taken in any order.

Examples.

(1) All B is A,
No C is not-B.

Here the two premisses are not in any particular valid mood, and seem to involve the fallacy of four terms. But, by permuting the second premiss, we obtain the following syllogism in *Barbara*:—All B is A; all C is B; ∴ all C is A.

(2) No C is not-B,
No B is not-A.

Here the two premisses are negative, and do not seem to justify any conclusion whatever. But, by permuting both, we get the following syllogism in *Barbara*:—All C is B; all B is A; ∴ all C is A, the first being the minor and the second the major premiss.

(3) No A is B,
No not-B is C.

Converting the first premiss, and permuting the converse of the second, we obtain the following valid syllogism in *Celarent*:—'No B is A; all C is B; ∴ no C is A.'

(4) 'No metal is a compound substance,
Gold is not a non-metal.'

By permuting the first and the second premiss, we get the following syllogism in *Barbara*:—"Every metal is an elementary (not-compound) substance; gold is a metal; therefore gold is an elementary substance."

Examples for Solution.

I.—Supply the premiss suppressed in the following:—

(1) Iron is a metal because it conducts heat and electricity.
(2) Gold is a noble metal because it does not rust.
(3) Air is material because it has weight.
(4) Air is a gas because it is not liquid or solid.
(5) This idea is real because it agrees with the external thing.
(6) Material things exist because they are the objects of my perception.
(7) A is the cause of B because it is its invariable antecedent.
(8) A must have a cause because it is a phenomenon.
(9) B must be a mineral because it has no signs of organization.
(10) C must be a plant because it has root and leaves.
(11) D can not be a bird because it has no feather.
(12) E is the effect of D because it invariably follows D.
(13) H can not be an acid because it has neither hydrogen nor oxygen.

II.—Supply premisses from which each of the following propositions can be inferred syllogistically:—

(1) Some elements are not metals.
(2) Gold is a metal.
(3) Gravity is a force.
(4) No metals are compounds.
(5) Only material bodies gravitate.
(6) Water is a compound body.

(7) Matter is indestructible.
(8) Electricity is not a form of matter.
(9) Silver is an element.
(10) All plants are organized.
(11) No crystal is organized.
(12) Some flowers are not odorous.
(13) Some animals have no power of locomotion.

III.—Draw the conclusion, if any, which follows from each of the following pairs of premisses:—

(1)—(a)	No not-A is B. No not-B is C.	(b)	No B is A. No C is not-B.
(2)—(a)	All B is not-A. No C is not-B.	(b)	No A is B. No C is not-B.
(3)—(a)	No B is A. Some C is not not-B.	(b)	No not-A is B. Some C is not not-B.
(4)—(a)	Some B is C. No not-A is B.	(b)	All A is B. All C is not-B.
(5)—(a)	No not-B is C. No B is A.	(b)	No not-C is B. No not-B is A.

(6) All metals conduct heat; all metals conduct electricity.
(7) All birds are oviparous; all birds cannot fly.
(8) Every feeling is a mental phenomenon; every feeling is not a sensation.
(9) If the rays of light reach the eye, a sensation is produced; if a sensation is produced, it is accompanied by a perception.
(10) Every sensation is accompanied by a perception; a sensation is sometimes produced internally without any external object.
(11) Every chemical union is accompanied by the evolution of heat; a chemical union is sometimes accompanied by the evolution of light.
(12) If two substances are rubbed together, heat is produced; if two substances are struck against each other, heat is produced.
(13) If this gas is carbonic dioxide, it will produce turbidity in a solution of lime-water; it does produce turbidity in that solution of lime-water.

(14) This substance is an element; an element is either a metal or a non-metal.

(15) A material body is either solid, liquid, or gaseous; this body is not gaseous.

(16) None but animals are sentient beings; all plants are insentient beings.

(17) Only material bodies gravitate; light does not gravitate.

(18) None but elements are metals, oxygen and chlorine are non-metals.

CHAPTER VI.

OF TRAINS OF SYLLOGISTIC REASONING.

§ 1. A Train of Syllogistic Reasoning is a combination of two or more syllogisms so connected with one another as to establish a single conclusion. When each of the component syllogisms is fully expressed, it has either of these two typical forms:

(1) That in which the single conclusion is stated last, and the conclusion in one syllogism forms a premiss in the next.

(2) That in which the single conclusion is stated first, and a premiss in one syllogism forms the conclusion in the next, or both premisses form conclusions in two distinct syllogisms.

First Form.	*Second Form.*
(1) All A is B ... (minor),	(1) All A is E ... (conclusion),
All B is C ... (major),	∵ All D is E ... (major),
∴ All A is C ... (conclusion),	All A is D ... (minor),
(2) All A is C ... (minor),	(2) All A is D ... (conclusion),
All C is D ... (major),	∵ All B is D ... (major),
∴ All A is D ... (conclusion),	All A is B ... (minor),
(3) All A is D ... (minor),	(3) All D is E ... (conclusion),
All D is E ... (major),	∵ All C is E ... (major),
∴ All A is E ... (conclusion).	All D is C ... (minor).

In the example of the first form the single conclusion is "All A is E" stated last, and the conclusion of the first syllogism is a premiss in the second, and the conclusion of the second a premiss in the third.

In the example of the second form, the single conclusion is the same (All A is E), but it is stated first, and the two premisses of the 1st syllogism form the conclusions in the 2nd and 3rd, *i.e.*, are proved by them.

The first syllogism in the first form is called a Prosyllogism in relation to the 2nd, and the 2nd in relation to the 1st is called an Episyllogism; that is, a Prosyllogism is a syllogism in a train of reasoning, whose conclusion forms a premiss in another, and an Episyllogism is a syllogism which has for one of its premisses the conclusion of another. These two terms are relative, and the same syllogism may be a prosyllogism in relation to one, and an episyllogism in relation to another. For example, the 2nd syllogism stands in the twofold relation to the 3rd and the 1st respectively.

In the example of the second form, the 1st syllogism is an episyllogism in relation to the 2nd and 3rd, and both these are prosyllogisms in relation to the 1st.

The first form is called Synthetic, Progressive, or Episyllogistic, because the advance in the reasoning is from a prosyllogism to an episyllogism, from certain premisses to the conclusion which follows from them. The second form is called Analytic, Regressive, or Prosyllogistic, because the advance in the reasoning is from an episyllogism to a prosyllogism, from a conclusion to the premisses which prove it.

§ 2. The synthetical train of syllogistic reasoning gives rise to the Synthetical Method, and the analytical train of syllogistic reasoning to the Analytical Method in Deductive Logic.

In the Synthetical Method we start with certain principles as premisses; and by comparing and combining them in various ways, we deduce the conclusions which follow necessarily from them. In the Analytical Method, on the contrary, we start with the conclusions, and proceed regressively to the principles from which they follow deductively. It is by the former method that Euclid proves his propositions; he starts with the axioms, postulates, and definitions as premisses, and proves *progressively* the propositions which follow from them.

§ 3. An episyllogistic or synthetic train of reasoning in which all the conclusions, except the last, are suppressed, is called a *Sorites*. Thus, omitting the conclusions of the 1st two syllogisms, and consequently also the minor premisses of the 2nd and 3rd in the example given above, we get a Sorites of the following form:—

All A is B,
All B is C,
All C is D,
All D is E,
∴ All A is E,

in which the conclusion of the prosyllogism forms the minor premiss in the next episyllogism. This is called the *Aristotelian Sorites*. When the conclusion of the Prosyllogism forms, on the other hand, the major premiss in the next Episyllogism, we have a sorites of a different form, called, after its discoverer, the *Goclenian Sorites*. In the fully expressed form the corresponding train of syllogistic reasoning is as follows:—

1.	All B is C	...	(major premiss),
	All A is B	...	(minor premiss),
∴	All A is C	...	(conclusion),
2.	All A is C	...	(major premiss),
	All D is A	...	(minor premiss),
∴	All D is C	...	(conclusion),
3.	All D is C	...	(major premiss),
	All E is D	...	(minor premiss),
∴	All E is C	...	(conclusion).

Suppressing all the conclusions except the last, and consequently also all the major premisses except the first, we have the following *Goclenian Sorites*:—

All B is C,
All A is B,
All D is A,
All E is D,
∴ All E is C.

Taking the following train of syllogistic reasoning :—

(1)	All D is E	...	(major premiss),
	All C is D	...	(minor premiss),
	∴ All C is E	...	(conclusion),
(2)	All C is E	...	(major premiss),
	All B is C	...	(minor premiss),
	∴ All B is E	...	(conclusion),
(3)	All B is E	...	(major premiss),
	All A is B	...	(minor premiss),
	∴ All A is E	..	(conclusion),

and suppressing all the conclusions except the last, and therefore also all the major premisses except the first, we have the following example of the *Goclenian Sorites* :—

All D is E,
All C is D,
All B is C,
All A is B,
∴ All A is E.

Both the Goclenian and the Aristotelian Sorites are abridged trains of syllogistic reasoning, and both are synthetic, progressive, or episyllogistic, the advance in the reasoning being from a prosyllogism to an episyllogism.

An *Epicheirema* is a prosyllogistic, analytical, or regressive train of reasoning with some of its premisses suppressed. It consists of a syllogism with a reason or reasons for one or both of its premisses being given. For example, the train of reasoning "All A is B ; and all C is A, because all C is D : therefore all C is B" is an Epicheirema, in which a reason is given for one premiss, and which may be thus fully expressed :—

(1)	All A is B	...	(major premiss),
	All C is A	...	(minor premiss),
	All C is B	...	(conclusion).

For the minor premiss the reason given is that 'All C is D.'

This with that premiss evidently constitutes an enthymeme, whose major premiss is suppressed, thus :—

(2)	All D is A	...	(the suppressed major premiss),
	All C is D	...	(the reason given),
∴	All C is A.		

In the following example reasons are given for both the premisses: "All A is B, because all A is G; all C is A, because all F is A; therefore all C is B." When fully expressed it consists of the following three syllogisms :—

(1)	All A is B	...	(major premiss),
	All C is A	...	(minor premiss),
∴	All C is B	...	(conclusion).

The major premiss is proved by an enthymeme, whose major premiss is suppressed :—

(2)	All G is B	...	(the suppressed major premiss),
	All A is G	...	(the reason given),
∴	All A is B	...	(conclusion).

The minor premiss is also proved by an enthymeme, whose minor premiss is suppressed :—

(3)	All F is A	...	(the reason given),
	All C is F	...	(suppressed minor),
∴	All C is A	...	(conclusion).

The *Epicheirema* is thus an abridged train of syllogistic reasoning, in which the argument proceeds analytically, from an episyllogism to a prosyllogism.

The analytic train of syllogistic reasoning which we have given at the beginning of this chapter may give rise to any of the following Epicheiremas by suppressing different premisses :—

(1)	All A is D,	∵ all A is B,
	All D is E,	∵ all C is E,
	∴ All A is E.	

(2) All A is D, ∵ all A is B,
All D is E, ∵ all D is C,
∴ All A is E.

(3) All A is D, ∵ all B is D,
All D is E, ∵ all C is E,
∴ All A is E.

(4) All A is D, ∵ all B is D,
All D is E, ∵ all D is C,
∴ All A is E.

In (1) the major premiss of the second syllogism and the minor of the third are suppressed.

In (2) the major premiss of the second and the major premiss of the third syllogism are suppressed.

In (3) the minor premiss of the second syllogism and the minor of the third are suppressed.

In (4) the minor premiss of the second syllogism and the major of the third are suppressed.

The different varieties of trains of syllogistic reasoning are shown in the following tabular view :—

TRAINS OF SYLLOGISTIC REASONING.

- Synthetic, Progressive, or Episyllogistic (from Pro- to Epi- syllogism).
  - Fully-expressed.
  - Abridged (or Sorites).
    - Aristotelian,
      All A is B,
      All B is C,
      All C is D,
      All D is E,
      ∴ All A is E.
    - Goclenian,
      All D is E,
      All C is D,
      All B is C,
      All A is B,
      ∴ All A is E.
- Analytic, Regressive, or Prosyllogistic (from Epi- to Pro- syllogism).
  - Fully-expressed.
  - Abridged (or Epicheirema).
    - Simple, with reason for one premiss only.
    - Double, with reasons for both the premisses.
    - Complex with reason for the reason

§ 4. Symbolical examples of Sorites with analyses :—

FIRST FIGURE.

Barbara.

Aristotelian.

I. All A is B,
All B is C,
All C is D,
∴ All A is D.

Analysis of I.

(1) All A is B ... (minor),
All B is C ... (major),
∴ All A is C ... (conclusion),

(2) All A is C ... (minor),
All C is D ... (major),
∴ All A is D ... (conclusion).

Goclenian.

I. All C is D,
All B is C,
All A is B,
∴ All A is D.

Analysis of I.

(1) All C is D ... (major),
All B is C ... (minor),
∴ All B is D ... (conclusion),

(2) All B is D ... (major),
All A is B ... (minor),
∴ All A is D ... (conclusion).

Darii.

II. Some A is B,
All B is C,
All C is D,
∴ Some A is D.

II. All C is D,
All B is C,
Some A is B,
∴ Some A is D.

N.B.—Analysis of II. is similar to that of I.

Celarent.

III. All A is B,
All B is C,
No C is D,
∴ No A is D.

Analysis of III.

(1) All A is B ... (minor),
All B is C ... (major),
∴ All A is C ... (conclusion),

(2) All A is C ... (minor),
No C is D ... (major),
∴ No A is D ... (conclusion).

III. No C is D,
All B is C,
All A is B,
∴ No A is D.

Analysis of III.

(1) No C is D ... (major),
All B is C ... (minor),
∴ No B is D ... (conclusion).

(2) No B is D ... (major),
All A is B ... (minor),
∴ No A is D .... (conclusion).

Ferio.

IV. Some A is B,
All B is C,
No C is D,
∴ Some A is not D.

IV. No C is D,
All B is C,
Some A is B,
∴ Some A is not D.

In the 1st figure one premiss only can be particular: the 1st in the Aristotelian and the last in the Goclenian; and only one premiss negative: the last in the former and the first in the latter. It should be observed that, when the conclusion is the same, the order of the premisses in one form is exactly the reverse of that in the other; that is, the conclusion being the same in both, the premisses in the Goclenian are those of the Aristotelian from the bottom upwards. This has given rise to the mistaken notion that the latter is progressive, while the former is regressive; but we have seen that both are equally progressive or episyllogistic. The order of the terms should also be noted. In the Aristotelian the predicate in one premiss becomes the subject in the next, while in the Goclenian the subject in one premiss becomes the predicate in the next.

SECOND FIGURE.

Aristotelian.	*Goclenian.*
V. All A is B,	V. No E is D,
All B is C,	All C is D,
All C is D,	All B is C,
No E is D,	All A is B,
∴ No A is E.	∴ No A is E.
Analysis of V.	*Analysis of V.*
(1) All A is B ... (minor),	(1) No E is D ... (major),
All B is C ... (major),	All C is D ... (minor),
∴ All A is C ... (conclusion),	∴ No C is E ... (conclusion),
(2) All A is C ... (minor),	(2) No C is E ... (major),
All C is D ... (major),	All B is C ... (minor),
∴ All A is D ... (conclusion),	∴ No B is E ... (conclusion),
(3) All A is D ... (minor),	(3) No B is E ... (major),
No E is D ... (major),	All A is B ... (minor),
∴ No A is E ... (conclusion).	∴ No A is E ... (conclusion).

In these examples only one syllogism is in the second figure; the others are in the first figure. In the Aristotelian the last, and in the Goclenian the first, are in the mood *Cesare* of the

~~second figure; all~~ the others are in the mood *Celarent* of the first figure.

It should be noted that in the Aristotelian Sorites the conclusion of a Prosyllogism becomes the minor premiss, while in the Goclenian it becomes the major premiss, in the next Episyllogism, throughout the whole train of reasoning. We shall conclude with an Aristotelian Sorites in the 3rd figure :—

VI. All A is B,
All B is C,
All C is D,
All A is E,
∴ Some D is E.

Analysis of VI.

(1) All A is B,
All B is C,
∴ All A is C,

(2) All A is C,
All C is D,
∴ All A is D,

(3) All A is D,
All A is E,
∴ Some D is E.

Here the 3rd Syllogism is in *Darapti* in the 3rd figure, and the others in *Barbara*.

§ 5. Questions and exercises.

1. Analyse and test the following trains of reasoning:—

(1) "Bucephalus is a horse; a horse is a quadruped; a quadruped is an animal; an animal is a substance: therefore Bucephalus is a substance."

(2) "If Harpagon be avaricious, he is intent on gain; if intent on gain, he is discontented; if discontented, he is unhappy; now Harpagon is avaricious: he is, therefore, unhappy."

(3) "Whatever promotes happiness is good; whatever perfects the soul promotes happiness: therefore whatever perfects the soul is good; misfortune which happens to the good, serves either to disci-

pline or to improve the soul: hence misfortune which befalls the good is good."

(4) "Sentient beings seek happiness; all finite beings are sentient; all men are finite beings; Caius is a man: therefore he seeks happiness."

(5) "That which thinks is active; that which is active has strength: that which has strength is a substance; the soul thinks: therefore it is a substance."

(6) A is equal to B; B is equal to C; C is equal to D; D is equal to E: therefore A is equal to E.

(7) A is greater than B; B is greater than C; C is greater than D; D is greater than E: therefore A is greater than E.

(8) A is the cause of B; B is the cause of C; C is the cause of D; D is the cause of E: therefore A is the cause of E.

(9) A lies above B; B lies above C; C lies above D: therefore A lies above D.

(10) A co-exists with B; B co-exists with C; C co-exists with D: therefore A co-exists with D.

(11) A is a mark of B; B is a mark of C; C is a mark of D: therefore A is a mark of D.

(12) If a gas is heated, its temperature rises; if its temperature rises, its elastic force increases; if its elastic force increases, the pressure on the walls of the containing vessel increases: therefore if a gas is heated, the pressure on the walls of the containing vessel increases.

2. Analyse the demonstration of the 20th Proposition in Todhunter's Euclid, p. 23, into the constituent syllogisms.

3. Prove both synthetically and analytically the 18th Proposition of Euclid, Book I, Todhunter, p. 22.

4. Analyse into fully-expressed syllogisms both the construction and the demonstration of the 32nd Proposition of Euclid, Book I.

5. Distinguish between the Analytical Method in Deductive Logic and Analysis as employed in Geometry.

CHAPTER VII.

Of Fallacies.

§ 1. I.—A General Outline.

A Fallacy, in the proper sense of the term, is a transgression of a rule of inference. A fallacious reasoning is, in fact, an apparent reasoning involving the breach of some rule or other of the various kinds of inference. Thus there are as many different kinds of Fallacies as of Reasoning or Inference.

The breach of the laws of Inductive Reasoning gives rise to the Fallacies of Induction with which we have nothing to do here. The breach of the rules of Deductive Inference gives rise, first, to the Fallacies of Immediate Inference, when the rule transgressed is a rule of Immediate Inference, and, secondly, to the Fallacies of Mediate Inference, when the rule violated is one of Syllogism or of any other kind of Mediate Deductive Reasoning. Thus we have the following classes of Fallacies in the sense we have defined above:—

INFERENTIAL FALLACIES.

Inductive (arising from the transgression of the canons of inductive reasoning).

Deductive (arising from the transgression of the rules of deductive reasoning).

Fallacies of Immediate Inference.

Fallacies of Conversion.
Permutation.
Contraposition.
Opposition.
Subalternation.
Modal Consequence.
Change of Relation.

Fallacies of Mediate Inference.

I. Syllogistic:—
Undistributed Middle.
Illicit Process.
Four Terms.
&c. &c.

II. Non-Syllogistic, *e. g.*, Mathematical.

In a wider sense a Fallacy is a transgression of any logical rule whatever. In this sense we have in Deductive Logic the Fallacies or Faults of Division and Definition; and in Inductive Logic those of Classification, Hypothesis, &c. The violation of the rules to which every logical division and definition ought to conform gives rise to the faults of division and definition, such as cross division, incomplete division, definition by accidental qualities, &c. To this class belong also the fallacies arising from ambiguity in language, such as those of Ambiguous Middle, of Division, Composition, &c. These are transgressions of the logical rule that our thoughts should be expressed and reasonings conducted in clear and unambiguous language.

NON-INFERENTIAL LOGICAL FALLACIES.

- Those usually treated in Deductive Logic.
  - Those arising from the transgression of the rules of Definition and Division.
    - Faults of Definition:—
      - (1) Description or definition by *accidental* qualities; redundant definition.
      - (2) Too narrow or too wide definition.
      - (3) The *circle* in definition, or definition by synonyms.
      - (4) Obscure, figurative, and ambiguous definitions.
      - (5) Negative definition.
    - Faults of Division: –
      - (1) Physical Partition and Metaphysical Analysis.
      - (2) Cross Division.
      - (3) Incomplete or Overcomplete (too narrow or too wide) Division.
      - (4) Overlapping Division.
  - Those arising from ambiguous language, called *Semi-logical*:—
    - Ambiguous Middle.
    - Fallacy of Composition.
    - " Division.
    - " Accident.
    - &c. &c.
- Those usually treated in Inductive Logic, arising from the transgression of the rules of Classification, Hypothesis, Nomenclature, &c.

In the widest sense, the word *fallacy* may be taken to mean an error of any kind, whether of Intuition, Perception, Observation, Division, Definition, Inference, &c. In this sense it includes, besides those mentioned above, the fallacies of Irrelevancy or Irrelevant Conclusion, technically called *Ignoratio Elenchi*, of *Petitio Principii* (begging the question), of False Premiss, and also those which Mill calls Fallacies of Simple Inspection, or of Erroneous First Principles and Axioms.

NON-LOGICAL OR MATERIAL FALLACIES.

Premiss unduly assumed.

Irrelevant conclusion or *Ignoratio Elenchi* (the argument or conclusion not to the point).

Premisses depending on the conclusion, *Petitio Principii*.

Premiss false or unsupported; *Non causa pro causa*; Erroneous First Principles and Axioms, &c. &c.

Premiss the same as the conclusion, the argument in a circle.

Premiss unfairly implying the conclusion.

Fallacies of appealing to the passions:
Argumentum ad hominem,
" *populum*,
" *verecundiam*.

Fallacy of shifting ground, &c.

§ 2. II.—Fallacies in Deductive Logic.

It is not necessary that we should describe and explain in detail each of the fallacies mentioned above, for most of them have been already made evident in explaining and illustrating the rules. In the following pages we shall notice and illustrate the more frequent and important kinds only.

A.—Logical Fallacies.

1. *Inferential.*

(1)—Fallacies of Immediate Inference.

In Conversion the most frequent fallacy is the simple conversion of A: 'All A is B, ∴ All B is A,' 'If A is, B is, ∴ If B is, A is.' The inference is, of course, fallacious, and violates the rule of conversion, viz., that no term should be distributed in the converse which was not distributed in the convertend; and the valid inference is 'Some B is A,' 'In some cases if B is, A is.' The simple conversion of O is also fallacious for the same reason: 'Some A is not B, ∴ Some B is not A.' The conversion of O into 'Some not-B is A' is not admissible, because it violates the first rule of conversion, viz., that the subject and the predicate of the convertend should be, respectively, the predicate and the subject in the converse.

In Obversion, Æquepolence, or Permutation the following are fallacious:—

(1) All A is B; ∴ All not-A is not-B.
(2) All metals are elements;
∴ All not-metals are not-elements.
(3) Cold is agreeable;
∴ Heat is disagreeable.
(4) Virtue will be rewarded;
∴ Vice will be punished.

In Contraposition the following are fallacious:—

(1) No A is B; ∴ All not-B is A.
(2) No man is perfect;
∴ All imperfect beings are men.
(3) Some A is B; ∴ Some not-B is A.
(4) Some elements are metals;
∴ Some not-metals are elements.

In Opposition the following are fallacious:—

(1) 'All plants are flowerless' is false;
∴ 'No plants are flowerless' is true.

(2) 'All philosophers are poets' is false;
∴ 'No philosophers are poets' is true.
(3) 'Some plants can move' is true;
∴ 'Some plants cannot move' is true.
(4) 'Some elements are metals' is true;
∴ 'Some elements are not metals' is true.
(5) 'Some men are wise' is true;
∴ 'Some men are not wise' is false.

§ 3. (2)—Fallacies of Syllogistic Inference.

These arise from the transgression of the syllogistic rules. Everyone of them is *ultimately* a breach of some one or other of the fundamental principles of Deductive Logic, and *proximately* of the general syllogistic rules, or of the special rules for each figure. Regarded as transgressions of the nine general syllogistic rules we have given in Part III. Chap. III. the fallacies are as follows:—

(1) The Fallacy of *Four Terms*, arising from the transgression of the 1st rule.

(2) The Fallacy of *Four Premisses*, arising from the violation of the 2nd rule.

(3) The Fallacy of *Undistributed Middle*, arising from the breach of the 3rd rule.

(4) The Fallacy of *Illicit Process*, arising from the transgression of the 4th rule: of the *Major Term*, when this term is distributed in the conclusion and not in the premiss; and of the *Minor Term*, when this term is distributed in the conclusion and not in the premiss.

(5) The Fallacy of *Negative Premisses*, arising from the violation of the 5th rule.

(6) Fallacies also arise from the transgression of the 6th, 7th, 8th, and 9th rules, and belong to one or other of the fallacies mentioned above.

The most important of the fallacies under this head are those of Undistributed Middle and Illicit Process. Of these we shall give the following examples:—

1. The virtuous are happy,
The wealthy are happy;
∴ The wealthy are virtuous.

Undistributed Middle, because the middle term being the predicate in the two affirmative premisses, is not distributed.

2. All material bodies are extended,
Shadows are extended;
∴ Shadows are material bodies.

Undistributed Middle.

3. Whatever thinks exists,
Matter does not think;
∴ Matter does not exist.

Illicit Process of the Major Term,

which being the predicate in the affirmative major premiss, is undistributed, but which is distributed in the conclusion, being the predicate in a negative proposition.

4. All material bodies have weight,
All material bodies are extended;
∴ All extended things have weight.

Illicit Process of the Minor Term,

which is distributed in the conclusion, but not distributed in the minor premiss.

5. All men are mortal,
All men are rational;
∴ All rational beings are mortal.

Illicit Process of the Minor Term.

6. All metals conduct heat and electricity,
All metals are elements;
∴ All elements conduct heat and electricity.

Illicit Process of the Minor Term.

7. All Hindus are Aryans,
The Persians are not Hindus;
∴ The Persians are not Aryans.

Illicit Process of the Major Term.

2. *Non-Inferential.*

§ 4. (1)—Semi-logical Fallacies.

These arise from ambiguous language. If a term is ambiguous, it is really equivalent to two, and there is thus the fallacy of *four terms.* In a fallacy of this kind, it is the middle term that is generally ambiguous, giving rise to what is called the fallacy of *ambiguous middle.* In some cases, the middle term is taken distributively in the major premiss, and collectively in the minor; in some it is taken collectively in the major and distributively in the minor premiss. In the former, we have the Fallacy of *Composition,* and in the latter the Fallacy of *Division.* We shall now give a few examples of each of these varieties:—

1. An organized body is either a plant or an animal; a nation is an organized body: therefore a nation is either a plant or an animal. Here the word *body* is ambiguous.

2. Light is a mode of motion; feather is light: therefore feather is a mode of motion. Here the double meaning of the word *light* is obvious.

3. "All cold is to be expelled by heat; this person's disorder is a cold: therefore it is to be expelled by heat." Here the word *cold* is ambiguous: in the first premiss it means a low degree of heat or the sensation of coldness, and in the second a particular bodily disorder.

4. "Projectors are unfit to be trusted; this man has formed a project: therefore this man is unfit to be trusted." Here *projector* and *formed a project* do not mean the same thing.

5. "To be acquainted with the guilty is a presumption of guilt; this man is so acquainted: therefore we may presume that he is guilty." Here the phrases '*presumption of guilt*' and '*presume that he is guilty*' have different significations.

6. "All the angles of a triangle are equal to two right angles, ABC is an angle of a triangle; ∴ ABC is equal to two right angles," is a Fallacy of *Division;* for the middle term is taken *collectively* in the major and *distributively* in the minor premiss.

7. "Five is one number; three and two are five: therefore three and two are one number," is also a Fallacy of *Division.*

8. "Three and two are two numbers; five is three and two: therefore five is two numbers," is a Fallacy of *Composition;* for the middle term is taken *distributively* in the major premiss, and *collectively* in the minor.

9. "All the angles of a triangle are less than two right angles, ABC, ACB, and BAC are all the angles of a triangle; ∴ they are less than two right angles."

Here the word *all* is ambiguous. In the major premiss the term '*all the angles of a triangle*' is taken *distributively* to mean any angle. In the minor premiss, it is doubtful whether it is taken collectively or distributively. If it is taken collectively, the argument involves the Fallacy of *Composition.* If it is taken distributively, the argument is valid.

10. "I can afford to buy these books. I can afford to buy these pictures. I can afford to buy these statuettes. The books, the pictures, and the statuettes are all that I, at present, wish to purchase. I can, therefore, buy everything that I want to buy." This is a Fallacy of *Composition;* 'these books,' 'these pictures,' and 'these statuettes' are taken *distributively* or separately in the first premiss, and *collectively* or jointly in the second.

11. The Fellows of the Royal Society have made the greatest discoveries in Science; A, B, and C are Fellows of the Royal Society; therefore A, B, and C have made the greatest discoveries in Science. This is a Fallacy of *Division.*

The next fallacy under this head is the Fallacy of *Accident,* which consists in taking a term *simply* or without any condition in one premiss, and as *modified by certain accidents* or as under certain circumstances in the other. For example, "What is bought in the market is eaten, raw meat is bought in the market; therefore raw meat is eaten." In the minor premiss the middle term, *bought in the market,* is taken *simply,* while in the major premiss it must be understood as *modified by certain accidents* or qualities not present in the other. There are, in fact, two middle terms, one 'bought in the market' without anything understood

after it, and the other 'bought in the market' with some such phrase as 'and cooked at home' understood after it. The Fallacy of Accident as defined above includes both the *Fallacia a dicto simpliciter ad dictum secundum quid* and the *Fallacia a dicto secundum quid ad dictum simpliciter*,—that is, both the fallacy of arguing from a simple statement to a statement under a certain condition, and the fallacy of arguing from a statement under a certain condition to a simple statement. There is another fallacy of a similar nature, in which the reasoning proceeds from a statement under a certain condition to a statement under a different condition. All the three forms of the Fallacy of Accident are due to the *ambiguity of language*, and may be easily avoided by stating the meaning of the propositions in clear and unambiguous language.

In conclusion, it appears that all the different kinds of Semi-logical Fallacies arising from ambiguous language are really different forms of the fallacy of Ambiguous Middle, and, in rare cases, of Ambiguous Extreme. In some the middle term is in itself ambiguous, having two meanings in the same form or in different forms or parts of speech. In others the ambiguity arises from some of the words being ambiguous, or from the grammatical structure of the sentence being ambiguous. But an ambiguous term, whatever be the source of its ambiguity, is really equivalent to two terms; and all the forms of ambiguous middle and of ambiguous extreme are really transgressions of the first syllogistic rule, that is, they are all *fallacies of four terms*.

§ 5. (2) The fallacies or faults arising from the violation of the rules of Logical Division and Definition have been already explained under those subjects, and do not require any separate treatment here.

B.—Non-Logical or Material Fallacies.

These fallacies do not properly belong to Deductive Logic, as they are concerned about the subject-matter of reasoning. The more important of them are: (1) the *Petitio Principii*, including

the 'Argument in a Circle,' and 'Begging the Question'; (2) the Falsity of Premiss; and (3) the *Ignoratio Elenchi*, or the Fallacy of Irrelevancy, or, as it is sometimes called, the Irrelevant Conclusion.

§ 6. (I)—Of the *Petitio Principii*.

This fallacy in its simplest form occurs when a proposition is proved by another proposition, and this other is again proved by the first. For example, 'A is, because B is; and B is, because A is.' Here the conclusion is proved by the premiss, and the premiss by the conclusion; and the fallacy is quite evident, and consists really in proving 'A is' by 'A is,'—the same by the same, *idem per idem*.

In the following example, the major premiss of the 1st syllogism is proved by the 2nd, and the major premiss of the 2nd by the 1st syllogism:—

I. 1. M is P,	2. S is P,
S is M;	M is S;
∴ S is P.	∴ M is P.

Here 'S is P' is proved by a syllogism whose major premiss is 'M is P,' and this premiss is proved by a syllogism whose major premiss is 'S is P.' Thus, 'S is P' is proved with the aid of 'M is P,' and 'M is P' is proved with the aid of 'S is P': wherefore 'S is P' is proved by 'S is P.' In this also the fallacy is almost quite evident. But if the two syllogisms here placed one after the other were, respectively, the first and the last of a long train of reasoning, it would not be so easy to detect the fallacy. And this difficulty is still further increased partly by the difference of language in which the same proposition may occur in different parts of the train, and partly by the omission of many intervening syllogisms. For example—

II. 1. A is B,	3. A is D,
B is C;	D is E;
∴ A is C.	∴ A is E.
2. A is C,	4. A is E,
C is D;	E is B;
∴ A is D.	∴ A is B.

In this train of reasoning the final conclusion in the 4th syllogism is the same as the minor premiss of the 1st, that is, this premiss is proved by the 4th syllogism. But how is this final conclusion established? By using as a premiss the proposition 'A is E,' which has been itself proved by taking the final conclusion 'A is B' as a premiss in the first syllogism. Thus the final conclusion is really established by taking itself as a premiss in a part of the train of reasoning.

In the 1st syllogism, 'A is C' is proved by taking 'A is B' as a premiss.

In the 2nd, 'A is D' is proved by taking 'A is C' as a premiss, and therefore by indirectly taking 'A is B' as a premiss.

In the 3rd, 'A is E' is proved by taking 'A is D' as a premiss, and therefore by taking indirectly 'A is B' as an ultimate premiss.

In the 4th, 'A is B' is proved by taking 'A is E' as a premiss, and therefore by taking indirectly 'A is B' as an ultimate premiss. That is, 'A is B' is proved by 'A is B.'

Or the fallacy may be exposed thus:—A is C, because A is B; and A is B, because A is E (4th syllogism), and A is E, because A is D (3rd syllogism), and A is D, because A is C (2nd syllogism), therefore A is B, because A is C. Thus 'A is B' is proved by 'A is C,' and 'A is C' is proved by 'A is B.' Here the use of the symbols has enabled us to detect the fallacy easily; but if the language of the last syllogism were different from that of the first, and if, moreover, some of the intervening syllogisms were suppressed, the train being much longer than that represented above, it would not be so easy to detect the fallacy, and expose it by analysing the whole train.

The *Petitio Principii* in the *stricter* sense may, then, be defined as a fallacy in which the conclusion is proved by means of itself, or in which the conclusion is the same as one of the premisses. In the *wider* sense it includes also those fallacies in which the conclusion follows from, or is presupposed by, one premiss *independently* of the others. For example—

III. All men are mortal,
Those who are mortal are not immortal;
∴ No man is immortal.

In order to prove the conclusion 'No man is immortal,' two premisses are advanced, and the argument is apparently stated in the form of a syllogism; but the conclusion really follows immediately from, or is presupposed by, the first or minor premiss 'All men are mortal,' which *obverted* gives the conclusion directly.

In the stricter sense, the *Petitio Principii* is called the *Argument in a Circle* because the final conclusion is the same as the first premiss, because the reasoning coming back whence it started, completes a circle. In the wider sense, including all forms, it is called *Begging the Question*, because it begs or surreptitiously takes for granted a proposition which is identical in meaning with, or is a consequence of, the very proposition to be proved.

§ 7. (2) Of the Falsity of Premiss.

The next fallacy under this head is the Falsity of Premiss. This fallacy occurs when one of the premisses is false; when something is regarded as a cause of an event, which is really not the cause, which is either merely a sign or an antecedent of it. It is also called *Non causa pro causa*, the assuming as a cause that which is not a cause, and *Post hoc ergo propter hoc*, or after this, and therefore on account of, or caused by, this.

Whately thus distinguishes the *Petitio Principii* from *Non causa pro causa:* "Let the name then of 'petitio principii' (*begging the question*)," he says, "be confined to those cases in which one of the Premisses either is manifestly the same in sense with the Conclusion, or is actually proved from it, or is such as the persons you are addressing are not likely to know, or to admit, except as an inference from the Conclusion; as, for example, if any one should infer the authenticity of a certain history, from its recording such and such facts, the reality of which rests on the evidence of that history. All other cases in which a Premiss (whether the expressed or the suppressed

one) has no sufficient claim to be admitted, I shall designate as the *Fallacy of undue assumption of a Premiss*[1]."

Whately gives the following as an example of "the Argument in a Circle":—"Some mechanicians attempt to prove (what they ought to have laid down as a probable, but doubtful, hypothesis) that every particle of matter gravitates equally: 'why?' 'because those bodies which contain more particles ever gravitate more strongly or are heavier'; but (it may be urged) those which are heaviest are not always more bulky? 'no, but still they contain more particles, though more closely condensed'; 'how do you know that?' 'because they are heavier'; 'how does that prove it?' 'because all particles of matter gravitating equally, that mass which is specifically the heavier must needs have the more of them in the same space[2].'"

There is a smaller circle in the following instance: "If any one argues that you ought to submit to the guidance of himself, or his leader, or his party, &c., because these maintain what is right; and then argues that what is so maintained is right, because it is maintained by persons whom you ought to submit to, and that these are himself and his party[3]."

The fallacy of *Non causa pro causa* occurs when a sign is mistaken for a cause, or whenever the relation of cause and effect is reversed, the effect being regarded as the cause, and the cause as the effect, or when a premiss assumed is false. For instance, "A great deal of money in a country is a pretty sure proof of its wealth; and thence has been often regarded as a cause of it; whereas in truth it is an effect." "So also exposure to want and hardship in youth has been regarded as a cause of the hardy constitution of those men and brutes which have been brought up in barren countries of uncongenial climate. Yet the most experienced cattle-breeders know that animals are, *cæteris paribus*, the more hardy for having been well fed and sheltered in youth; while early hardships, by destroying all the tender, ensure

[1] Whately's *Elements*, 9th Edition, p. 132.
[2] *Ibid.* p. 133. [3] *Ibid.* p. 133.

the hardiness of the survivors, which is the cause, not the effect, of their having lived through such a training. So, loading a gun-barrel to the muzzle and firing it does not *give* it strength; though it *proves*, if it escape, that it *was* strong[1]."

(3) Of the *Ignoratio Elenchi.*

(This fallacy occurs whenever in any debate or discussion the conclusion arrived at, or the argument advanced, is not to the point: you wish to disprove or establish a certain proposition, and for this purpose you advance arguments which lead to a conclusion which is quite irrelevant to the subject at hand.) For example, you wish to prove that a certain doctrine is false; and instead of adducing facts or principles or both, which really disprove it, you dilate upon its consequences, upon the small number of its adherents, upon the moral qualities of its promulgators, and so forth. The way in which the Theory of Evolution is at the present day attacked by some, and defended by others, will furnish us with very apt illustrations of this fallacy. On the one hand, many popular speakers and writers attempt to refute it by arguments which have reference only to its consequences, to its appearance of absurdity, and to the prejudices and sentiments of the people; and, on the other, many of its defenders attempt to prove it by arguments which are no better than the former, having reference only to the high authority of the scientific men who believe in it, to their numerical strength, to the grandeur and beauty of the Theory, to the impossibility of the popular doctrine being true, and so forth. Both the opponents and the defenders of the Theory are equally guilty of the fallacy of *Ignoratio Elenchi*, inasmuch as they do not address themselves to the facts and principles really bearing upon the question.

Whately describes and illustrates the more important forms of this fallacy as follows:—

"It is evident," says Whately, "that *Ignoratio Elenchi* may be employed as well for the apparent refutation of your op-

[1] Whately's *Elements*, 9th Edition, p. 135.

ponents' proposition, as for the apparent establishment of your own; for it is substantially the same thing to *prove* what was not denied, or to *disprove* what was not asserted. The latter practice is not less common; and it is more offensive, because it frequently amounts to a personal affront, in attributing to a person opinions, &c., which he perhaps holds in abhorrence. Thus, when in a discussion one party vindicates, on the ground of expediency, a particular instance of resistance to Government in a case of intolerable oppression, the opponent may gravely maintain, that 'we ought not to do evil that good may come'—a proposition which of course had never been denied; the point in dispute being 'whether resistance in this particular case *were* doing evil or not.' Or again, (by way of disproving the assertion of the '*right* of private judgment in religion,' one may hear a grave argument to prove that 'it is impossible that every one could be *right in his judgment*.') In these examples, it is to be remarked that the fallacy of *Petitio Principii* is combined with that of *Ignoratio Elenchi;* which is a very common and often successful practice,—viz., the Sophist proves, or disproves, not the proposition which is really in question, but one which is so dependent on it as to proceed on the supposition that it is already decided, and can admit of no doubt; by this means his 'assumption of the point in question' is so indirect and oblique, that it may easily escape notice; and he thus establishes, practically, his conclusion, at the very moment he is withdrawing your attention from it to another question. For example, an advocate will prove, and dwell on the high *criminality* of a certain act, and the propriety of severely punishing it; assuming (instead of proving) the *commission*.

"There are certain kinds of arguments recounted and named by logical writers which we should by no means universally call Fallacies; but which *when unfairly* used, and *so far as they are* fallacious, may very well be referred to the present head; such as the *Argumentum ad hominem* (or personal argument), *Argumentum ad verecundiam*, *Argumentum ad populum*, &c., all of them regarded as contradistinguished from *Argumentum ad rem*

or *ad judicium*. These have all been described in the lax and popular language before alluded to, but not scientifically: the *Argumentum ad hominem*, they say, 'is addressed to the peculiar circumstances, character, avowed opinions, or past conduct of the individual, and therefore has a reference to him only, and does not bear directly and absolutely on the real question, as the *Argumentum ad rem* does'; in like manner, the *Argumentum ad verecundiam* is described as an appeal to our reverence for some respected authority, some memorable institution, &c., and the *Argumentum ad populum* as an appeal to the prejudices, passions, &c., of the multitude; and so of the rest[1]."

"The fallacy of Irrelevant Conclusion (*Ignoratio Elenchi*) is nowhere more common than in protracted controversy, when one of the parties having attempted in vain to maintain his position, *shifts his ground* as covertly as possible to another, instead of honestly giving up the point. An instance occurs in an attack made in the system pursued at one of our universities. The objectors finding themselves unable to maintain their charge of the present neglect (viz., in the year 1810) of Mathematics in that place (to which neglect they attributed the late *general decline* in those studies), shifted their ground, and contended that that University 'was never famous for mathematicians'; which not only does not establish, but absolutely overthrows, their own original assertion; for if it *never* succeeded in these pursuits, it would not have caused their late *decline*[2]."

§ 9. Besides the fallacies we have mentioned above, two more, namely, the *Non sequitur* and the *Fallacy of many questions*, are also given under the class of material fallacies. The first occurs when the conclusion does not in any way follow from the premisses, when, in fact, there is no logical connection between the two, anything being inferred from anything else. The second occurs when, by way of asking questions, certain assumptions are made in regard to certain things or persons: "In what subjects did you fail?" This question assumes

[1] Whately's *Elements*, pp. 141—142. [2] *Ibid.* pp. 143—4.

(1) that you appeared at an examination, and (2) that you failed in more than one subject; while the real fact might be just the reverse.

All these fallacies, as I have already said, do not properly belong to Deductive Logic. It is no part of Deductive Logic to ascertain whether a certain premiss is true or false, or whether a conclusion or an argument advanced by a party is irrelevant to the subject in debate. The *Petitio Principii*, indeed, would seem to be a fallacy of Deduction, inasmuch as the rules of Deductive Inference imply that a proposition can not be proved by means of itself, that a proposition, when inferred, must be inferred from others which are severally different from itself

§ 10. Exercises.

1. In testing an argument consisting of a single categorical syllogism, the following method should be followed:—

(i) Find the conclusion and note its subject and predicate which are, respectively, the minor and the major term of the syllogism.

(ii) Find the term which is not in the conclusion. It must be the middle term. (*a*) See if there be any other; if there is, then the argument involves the fallacy of *four terms*. (*b*) See if the middle term be ambiguous; if it is, then there is the fallacy of *ambiguous middle*. (*c*) See whether the middle term be distributed; if it is not, then there is the fallacy of *undistributed middle*.

(iii) Find the premiss which contains the minor term, and the premiss which contains the major term; and these two premisses are, respectively, the minor and the major premiss.

(iv) See if there be any term which is undistributed in either premiss, but distributed in the conclusion. If there is, then there is an *illicit process*.

If there be none of the above fallacies, then the argument is valid. To confirm this,

(v) Find the figure and mood of the syllogism, and see if the mood is a valid one in that figure.

2. In many cases the invalidity of an argument may be detected on mere inspection. For instance, when it contains two particular or two negative premisses, or when the middle term is not distributed, or when one of the premisses is negative and the Conclusion affirmative, or, lastly, when one of the premisses is particular and the conclusion universal.

3. The method described above seems, on the whole, to be the best. But there are of course other methods, which may also be applied to verify the result obtained by it or to test the argument independently. For example, the figure and the mood of the syllogism may be at once found; if the mood be a valid one in the particular figure, the syllogism will be valid. Or the figure being found, the syllogism may be tested by the canon or the special rules of that figure; if it conform to the canon or to the rules, it will be valid. Or the syllogism may be tested by the method of the comparison of the diagrams: if the conclusion follow in every case, it will be valid; if it do not follow in a single case, it will be invalid[1].

4. If an argument consists of more than one syllogism, that is, of a train of reasoning, it should be analysed into the constituent syllogisms; and each of them should be tested as described above. If any of the premisses be understood or suppressed, they should be supplied, and the constituent syllogisms fully expressed. In the case of Enthymemes, the suppressed premiss, whether true or false, should be supplied. In the case of Dilemmatic and other *mixed* arguments, they should be tested by their rules, and reduced to the categorical form. In the case of Extra-logical or Material fallacies, the student should be able to refer them to their respective classes and show where the fallacy lies.

Examples.

Test the following arguments:—

1. Every metal conducts heat; every metal conducts electricity: therefore every substance that conducts heat conducts electricity.

2. No minerals are plants; no plants are animals: therefore no minerals are animals.

3. All plants are organized; no crystals are plants: therefore no crystals are organized.

[1] Read also the directions given in Part III. Chap. v.

4. All birds are feathered; bats are not birds: therefore bats are not feathered.

5. All feathered animals are birds; bats are not birds: therefore bats are not feathered animals.

6. Only animals are sentient beings; fishes are animals: therefore fishes are sentient beings.

7. None but the Hindoos worship *Shiva;* all Bengalees are Hindoos: therefore all Bengalees worship *Shiva.*

8. All metals except one are solid; this substance is a metal: therefore it is solid.

9. Every object of thought is either an idea of sensation or an idea of reflection; matter is neither: therefore matter is not an object of thought.

10. Every element is either a metal or a non-metal; hydrogen is an element: therefore it is either a metal or a non-metal.

11. Fishes live in water; whales live in water: therefore whales are fishes.

12. Water is liquid; ice is water: therefore ice is liquid.

13. Plato is a philosopher; Plato approves of communism: therefore a philosopher approves of communism.

14. Aristotle believes in the immortality of the rational soul; Aristotle is the greatest intellect ever born: therefore the greatest intellect ever born believes in the immortality of the rational soul.

15. All poets are not imaginative, some philosophers are poets: therefore some philosophers are not imaginative.

16. "The Cretans are liars; A, B, C are Cretans: therefore A, B, C are liars."—Hamilton, Vol. III.

17. Every planet moves round the sun; the earth moves round the sun: therefore the earth is a planet.

18. Knowledge is power; perception is knowledge: therefore perception is power.

19. Cognition is a mental act; cognition is knowledge; knowledge is power: therefore power is a mental act.

20. "Whatever is dictated by nature is allowable; devotedness to the pursuit of pleasure in youth, and to that of gain in old age, are dictated by nature: therefore they are allowable."—Whately.

21. "That man is independent of the caprices of fortune who places his chief happiness in moral and intellectual excellence; a true

philosopher is independent of the caprices of fortune: therefore a true philosopher is one who places his chief happiness in moral and intellectual excellence."—Whately.

22. Give thanks unto the Lord; for he is good; for his mercy endureth for ever.

23. "Some objects of great beauty answer no other perceptible purpose but to gratify the sight; many flowers have great beauty; and many of them accordingly answer no other purpose but to gratify the sight."

24. "War is productive of evil; therefore peace is likely to be productive of good."—Whately.

25. "All that glitters is not gold; tinsel glitters: therefore it is not gold."—Whately.

26. If the rays of light reach the eye, or if the vibrations of sound reach the ear, a sensation is produced; but a sensation is not produced: therefore neither have the rays of light reached the eye, nor have the vibrations of sound reached the ear.

27. Electricity is neither a form of matter nor a form of energy; all material objects are either forms of matter or forms of energy: therefore electricity is not a material object.

28. If two oppositely electrified bodies be brought near, they attract each other; these two bodies repel: therefore they are not oppositely electrified.

29. If two similarly electrified bodies be brought near, they repel each other; these two bodies are not similarly electrified: therefore they do not repel each other.

30. The theory of evolution must be true because every scientific man worthy of the name believes in it.

31. A material body is either solid or fluid; this body is solid: therefore it is not fluid.

32. Every element is either solid or fluid; every element is not fluid: therefore every element is solid.

33. If a chemical union takes place, either heat or light is evolved; if oxygen and nitrogen are united in the proportions in which they exist in the atmospheric air, neither heat nor light is produced: therefore if oxygen and nitrogen are united in those proportions, no chemical union takes place.

34. If Darwin's theory of the origin of species be not true, every

species must be recognized as a special creation; but it is impossible that God should have created so many different species, when he could have easily evolved them all from a few: therefore Darwin's theory of the origin of species is true.

35. Plato is the father of Idealism; Plato is the founder of Political Philosophy: therefore the father of Idealism is the founder of Political Philosophy.

36. "The volume of a body diminishes when it is cooled, because the molecules then become closer."—Ganot's *Popular Physics*.

37. "Impenetrability and extension might be more aptly termed essential attributes of matter, since they suffice to define it."—Ganot.

38. "The struggle for existence reaches even to these little creatures, for they devour still smaller ones."—Ganot.

39. "Since the volume of every body may be diminished, we conclude that all bodies possess physical pores."—Ganot.

40. "No absolute rest is known in the universe; for the earth and the other planets rotate about the sun and about their own axis; and therefore all the parts composing them share this double motion."—Ganot.

41. "Whenever a body is heated, its volume increases, because its molecules are driven apart."—Ganot.

42. Matter is extended because it is impenetrable; and it is impenetrable because every part of it occupies a certain portion of space.

43. "A negro is a man: therefore he who murders a negro murders a man."—Whately.

44. "Meat and drink are necessaries of life; the revenues of Vitellius were spent on meat and drink: therefore the revenues of Vitellius were spent on the necessaries of life."—Whately.

45. "He who calls you a man speaks truly; he who calls you a fool, calls you a man: therefore he who calls you a fool speaks truly."

46. "Warm countries alone produce wines; Spain is a warm country: therefore Spain produces wines."—Whately.

47. "What we eat grew in the fields; loaves of bread are what we eat: therefore loaves of bread grew in the fields."—Whately.

48. Matter is impenetrable because it is extended; and it is extended because every atom of it, however small in dimensions, must occupy some little space.

49. "We are conscious of one mental state only as we contradistinguish it from another."—Hamilton's *Metaphysics*, Vol. I.

50. "We are conscious of an external world only as we are conscious of it as distinct from others."—Hamilton, Vol. I.

51. Truly we serve, because freely we love.

52. "A judgment is a simple act of mind, for every act of mind implies judgment."—Hamilton, Vol. I.

53. "Every mental phenomenon is either an act of knowledge, or only possible through an act of knowledge, for consciousness is a knowledge—a phenomenon of cognition."—Hamilton, Vol. I.

54. "Certain thoughts are universal, inasmuch as they arise under the same conditions in all men; they are necessary, because their genesis under these conditions is invariable."—Huxley's *Hume*, p. 86.

55. "For those who are bent on cultivating their minds by diligent study, the incitement of Academical honours is unnecessary; and it is ineffectual for the idle, and such as are indifferent to mental improvement: therefore the incitement of Academical honours is either unnecessary or ineffectual."—Whately.

56. "Those who hold that the insane should not be punished ought in consistency to admit also that they should not be threatened; for it is clearly unjust to punish any one without previously threatening him."

57. "If he pleads that he did not steal the goods, why, I ask, did he hide them, as no thief ever fails to do?"

58. "'No one can maintain that all Republics secure good government who bears in mind that good government is inconsistent with a licentious press.' What premisses must be supplied in order to express the above reasoning in Ferio, Festino, and Ferison, respectively?"

59. "If all were capable of perfection, some would have attained it; but, none having done so, none are capable of it."

60. "As thought is existence, what contains no element of thought must be the non-existent."

61. "Since the laws allow everything that is innocent, and avarice is allowed, it is innocent."

62. "Timon being miserable is an evil-doer, as happiness springs from well-doing."

63. "You can not stand still either intellectually or morally; and, therefore, if you are not advancing in the one or the other or both respects, you must be falling back."

64. Nothing and pure being are identical, inasmuch as both are devoid of all qualities.

65. "Theft is a crime; theft was encouraged by the laws of Sparta: therefore the laws of Sparta encouraged crime."—Whately.

66. "Revenge, Robbery, Adultery, Infanticide, &c., have been countenanced by public opinion in several countries; all the crimes we know of are Revenge, Robbery, Adultery, Infanticide, &c.: therefore all the crimes we know of have been countenanced by public opinion in several countries."—Whately.

67. "Every hen comes from an egg; every egg from a hen: therefore every egg comes from an egg."—Whately.

68. "Switzerland is a Republic, and, you will grant, a more stable Power is not to be found; nor, again, is any political society more settled than the United States. Surely, then, Republican France can be in no danger of revolution."

69. "If a conclusion is more certain to be wrong where the reasoning is correct from premisses that are false, will not the best logician be the least safeguard in subjects where perfect certainty is unattainable?"

70. "No one should be punished if he is innocent; this man should not be punished: therefore he is innocent."

71. "Every rule has exceptions; this is a rule, and therefore has exceptions: therefore there are some rules that have no exceptions."

72. "If I am to pass this examination I shall pass whether I do my papers or not; and if I am not to pass, I shall not pass whether I do my papers or not: therefore it is no matter whether or not I do my papers."

73. "A necessary being cannot be the effect of any cause; for if it were, its existence would depend upon that of its cause and would be no longer necessary."

74. "Whatever is conditioned must depend on some cause external to itself; this world is conditioned by time and space: therefore this world depends upon some cause external to itself."

75. "Position we must evidently acknowledge to be relative, for we cannot describe the position of a body in any terms which do not express relation."—Maxwell's *Matter and Motion*, p. 84.

76. If the theory of evolution be true, man is descended from

the lower animals; if the theory of evolution be true, man is not a special creation: therefore if man is not a special creation, he is descended from the lower animals.

77. "The learned are pedants; A is a learned man: therefore A is a pedant."

78. "If it be fated that you recover from your present disease, whether you call in a doctor or not, you will recover; again, if it be fated that you do not recover from your present disease, whether you call in a doctor or not, you will not recover; but one or other of the contradictories is fated: therefore to call in a doctor is of no consequence."—*Vide* Hamilton, Vol. III. pp. 462, 464.

79. "Perception is a cognition or act of knowledge; a cognition is an immanent act of mind; but to suppose the cognition of any thing external to the mind would be to suppose an act of the mind going out of itself, in other words, a transeunt act; but action supposes existence, and nothing can act where it is not: therefore to act out of self is to exist out of self, which is absurd."—Hamilton's *Lectures*, Vol. II. p. 118.

80. "Mind and matter, it is said, are substances, not only of different, but of the most opposite natures; separated, as some philosophers express it, by the whole diameter of being; but what immediately knows must be of a nature correspondent, analogous to that which is known; mind cannot, therefore, be conscious or immediately cognizant of what is so disproportioned to its essence as matter."—Hamilton's *Lectures*, Vol. II. p. 120.

81. "The mind can only know immediately that to which it is immediately present; but as external objects can neither themselves come into the mind, nor the mind go out to them, such presence is impossible: therefore external objects can only be immediately known through some representative object."—Hamilton's *Lectures*, Vol. II. p. 122.

82. "The table, which we see, seems to diminish, as we remove farther from it; but the real table which exists independently of us suffers no alteration: it was, therefore, nothing but its image which was present to the mind."—Hume.

83. "Take, for example, the term *man*. Here we can call up no notion, no idea, corresponding to the universality of the class or term. This is manifestly impossible. For as *man* involves contradictory

attributes, and as contradictory attributes can not co-exist in one representation, an idea or notion adequate to *man* can not be realized in thought."—Hamilton, Vol. II. p. 297.

84. "The class *man* includes individuals, male and female, white and black, copper-coloured, tall and short, fat and thin, straight and crooked, whole and mutilated, &c., &c.; and the notion of the class must, therefore, represent all and none of these. It is, therefore, evident that we can not accomplish this; and this being impossible, we can not represent to ourselves the class *man* by any equivalent notion or idea."—Hamilton, Vol. II. p. 297.

85. "It is manifest, indeed, that man, so far as he is a man for the glory of God, must be an end unto himself, for it is only in the accomplishment of his own perfection that, as a creature, he can manifest the glory of his Creator."—Hamilton, Vol. I. p. 5.

86. "Consciousness supposes a contrast—a discrimination; for we can be conscious only inasmuch as we are conscious of something; and we are conscious of something only inasmuch as we are conscious of what that something is—that is, distinguish it from what it is not."—Hamilton, Vol. I.

87. "Energy can not exist except in connexion with matter. Hence, since in the space between the sun and the earth, the luminous and thermal radiations, which have left the sun and which have not reached the earth, possess energy, the amount of which per cubic mile can be measured, this energy must belong to matter existing in the interplanatory spaces, and since it is only by the light which reaches us that we become aware of the existence of the most remote stars, we conclude that the matter which transmits light is disseminated through the whole of the visible universe."—Maxwell's *Matter and Motion*, p. 93.

CHAPTER VIII.

FUNCTIONS AND VALUE OF THE SYLLOGISM.

§ 1. ACCORDING to Mill the syllogistic process is not the process according to which we reason. "'All inference," says he, "is from particulars to particulars: general propositions are merely registers of such inferences already made, and short formulæ for making more. The major premiss of a syllogism consequently is a formula of this description; and the conclusion is not an inference drawn from the formula, but an inference drawn according to the formula; the real, logical antecedent or premiss being the particular facts from which the general proposition was collected by Induction[1]." "The value, therefore, of the syllogistic form, and of rules for using it correctly, does not consist in their being the form and the rules according to which our reasonings are necessarily, or even usually, made; but in their furnishing us with a mode in which those reasonings may always be represented, and which is admirably calculated, if they are inconclusive, to bring their inconclusiveness to light. An induction from particulars to generals, followed by a syllogistic process from those generals to other particulars, is a form in which we may always state our reasonings if we please. It is not a form in which we *must* reason, but it is a form in which we *may* reason, and into which it is indispensable to throw our reasoning, when there is any doubt of its validity: though when the case is familiar and little complicated, and there is no sus-

[1] *Logic*, Vol. I. p. 221.

picion of error, we may, and do, reason at once from the known particular cases to unknown cases[1]."

The universal type of the reasoning process, according to Mill, is as follows:—"Certain individuals have a given attribute; an individual or individuals resemble the former in certain other attributes; therefore they resemble them also in the given attribute[2]." This type is not, however, conclusive like the syllogism from the mere form of the expression; but must, in every case, be examined by the canons and rules of Induction. For example, 'all men now living resemble those men who have heretofore died' in certain attributes; whether from their resemblance in these attributes we may infer also their resemblance in the attribute 'mortality' is a question of Induction, and must be determined by its canons. If we may infer this attribute of 'all men now living,' we may infer it also of all other individuals that resemble the men who have died in the same attributes. This process of inference admits of a division into two steps: (1) "That of ascertaining what attributes are marks of mortality, universally, *i.e.*, under all circumstances, and (2) whether any given individuals possess those marks."

Conformably to usage, the first step or process, namely, that of establishing the general proposition, is called Induction, and the second step in "the reasoning operation, which is substantially that of interpreting the general propositions," is called Deduction by Mill. Every process by which any thing is inferred respecting an unobserved case, consists similarly of an Induction followed by a Deduction. According to Mill, the syllogism is thus merely a process by which the real or complete meaning of a general proposition established by Induction is made explicit, and by which the validity of a reasoning is tested. It is, in other words, an *interpreter* of the general proposition and a *test* of reasoning. Its rules and canons are merely cautions against false reasoning. They merely help us in interpreting correctly the true meaning of general propo-

[1] *Logic*, Vol. I. pp. 227—8.

[2] *Ibid.* p. 232.

sitions, and in applying them to particular cases. In ordinary discourse the reasoning is never conducted nor stated in the syllogistic form; but whenever there is any doubt about its validity, we may, or rather we *must*, throw it into the syllogistic form, and if it admits of being so expressed, we may be perfectly sure of its being valid. The syllogistic is not, therefore, the process according to which we usually reason. The universal process of reasoning is, according to Mill, from some particulars to other particulars; and the syllogistic process is merely a *test* of the validity of this process.

§ 2. Nor, according to Mill, is the syllogistic mode of arguing a sound one. "For," says he, "it must be granted that in every syllogism, considered as an argument to prove the conclusion, there is a *petitio principii*. When we say, 'all men are mortal, Socrates is a man; therefore Socrates is mortal,' it is unanswerably urged by the adversaries of the syllogistic theory, that the proposition 'Socrates is mortal' is presupposed in the more general assumption 'All men are mortal'; that we cannot be assured of the mortality of all men, unless we are already certain of the mortality of every individual man, &c., &c.; that, in short, no reasonings from generals to particulars can as such prove anything; since from a general principle we can not infer any particulars but those which the principle itself assumes as known[1]."

Regarded as a mode of Probation, the syllogism involves, according to Mill, the fallacy of *petitio principii*, that is, the conclusion is presupposed by the major premiss. The proposition 'all men are mortal' can not be true, unless the conclusion 'Socrates is mortal' is true. The truth of the latter is presupposed by the former, or the former can not be true unless the latter is. When you have assumed the major, you have already taken for granted the conclusion. Thus the conclusion is not really proved by the premisses of the syllogism. It is, on the contrary, proved by those particular cases of observation which

[1] *Logic*, Vol. I. p. 210.

establish the general or major premiss. It is these that are alike the evidence of the major premiss and of the conclusion of the syllogism.

The syllogism is thus, according to Mill, neither the process according to which we reason, nor an argument which is sound and free from fallacy. Is it, then, altogether useless? No, says Mill, its proper function is to interpret a general proposition and apply it to particular cases, and its real value consists in being an infallible test of the validity of the true process of reasoning. This process is, according to Mill, from particulars to particulars in accordance with the laws and canons of Induction. But when an inference is drawn from some particulars to some other particulars, we can not be quite certain that the reasoning is valid unless it admits of being thrown into the syllogistic form. That is, if, from 'some particulars,' we can infer a general proposition, and if with this general as a major premiss, and with 'some other particulars,' as a minor, we can form a valid syllogism, then the reasoning is valid. If the general can not be inferred, and the syllogism can not be formed, then the reasoning is invalid. For example, the reasoning that "all things now living are mortal, because all men in past ages have died," is completed according to inductive methods; but it will not be valid, unless a general proposition "all men are mortal" can be inferred from the particular cases of men who have died in past ages, and unless 'all kings now living' are really referable to the class 'man,' that is, the validity of the reasoning which is actually and really conducted from particulars to particulars in accordance with the canons of Induction, may be tested by reducing it to the following syllogism: "all men are mortal, all kings now living are men; therefore all kings now living are mortal."

This view of the functions and value of the syllogism, first propounded by Mill, has been adopted by Sir John Herschel, Dr Whewell, Mr Bailey, Professor Bain, and others. It has, on the other hand, been strongly opposed by Mansel, Professor De Morgan, Dr James Martineau, and others.

§ 3. There are two essential points in Mill's view of the syllogism,—(1) that it is not the usual process of reasoning, (2) that it involves the fallacy of *petitio principii*.

On the first point Mill maintains, that the universal process of reasoning is from particulars to particulars; and on the second point, that the real proof of the conclusion is not the premisses of the syllogism, but the facts of observation and testimony on which the major premiss itself is founded. On these two points the following observations may be made:—

1. It is true that the syllogism is not the process by which we usually reason. But it is equally true that our usual reasonings will not be valid, and therefore not deserve the name, unless they are capable of being reduced to the syllogistic form. Mill seems to make a confusion between the business of Psychology and that of Logic. It is not the business of the latter to give an account of the various processes by which people reason correctly or incorrectly, but to give an account of the processes by which they *ought* to reason, and *must* reason if they wish to reason correctly. The former is the business of the Psychology of Reasoning, while the latter is the business of the Logic of Reasoning. Mill confuses these two, and makes both the business of Logic. Recognizing the distinction here drawn, it may be said that the syllogism is the type of all valid reasoning; for no reasoning will be valid, as Mill also allows, unless it can be thrown into the form of a syllogism. As a matter of fact, in daily life, men draw inferences in many different ways, but only those among them will be valid, and properly deserving of the name, which are capable of being ultimately reduced to the syllogistic form, the rest being nothing but suggestions of association, fancy, imagination, &c., wrongly called inferences[1].

§ 4. 2. Secondly,—Does the syllogism involve the fallacy of *petitio principii?* On this most important subject the following noteworthy remark by Dr James Martineau is well deserving of being quoted; and as the book in which it is con-

[1] *Vide* Appendix D.

tained is not usually accessible to students, I will give it in full :—

"From the embarrassment of this objection we may extricate ourselves at once by simply remembering that, in the nature of things, or in the sight of a perfect intellect, whose processes are unconscious of succession or delay, *all* reasoning must involve a *petitio principii*, the conclusion being already discerned on the first announcement of the premiss. Ratiocination itself becomes nugatory in presence of a mind seeing by intuition what others reach by sequence. As soon as we descend to a more tardy and limited intelligence, there will be some beliefs that are mediately reached: the same truths which to one being are contained within their *arche* (ἀρχὴ) are seen by another lying at some distance from it. The *petitio principii* is thus entirely relative to the state and range of the individual understanding, and cannot be established as a fault against an argument by merely showing that the inference *might* be thought already in the assumption, but only by showing that it *must* be. If Mr Bailey can convince us that it is impossible to conceive the proposition 'birds are warm-blooded' without simultaneously contemplating the particular case of the swallow, we will grant that the conclusion 'swallows are warm-blooded' is a new inference of *idem per idem*. But if not,—if the general law can be formed, and, as he allows, rationally formed, without the mind having ever encountered this special instance,—it is vain to pretend that the conclusion only repeats in part the thought contained in the premiss. This is, no doubt, true of the reasoner, who, to bring conviction, invents the syllogism in question: he selects his general rule precisely, *because* he foresees what it contains; but in using it, he assumes in his learners a different state of mind,—in which the law has been apprehended and the example has been missed. Whenever a teacher and a learner are engaged together, the arguments comprehended in the didactic process involve a *petitio principii* to the former, but not to the latter. Upon this difference, the consciousness in one man, the unconsciousness in another, of what, according to the laws of

thought, a given proposition may imply, depends persuasion. Mr Mill, we are aware, treats this doctrine with no respect, and calls Archbishop Whately to severe account for sanctioning it. 'When you admitted the major premiss,' contends Mr Mill, 'you asserted the conclusion; but, says Archbishop Whately, you asserted it by implication merely: this, however, can here only mean that you asserted it unconsciously; that you did not know you were asserting it; but if so, the difficulty revives in this shape,—ought you not to have known? Were you warranted in asserting the general proposition without having satisfied yourself of the truth of everything which it fairly includes? And if not, what then is the syllogistic art but a contrivance for catching you in a trap and holding you fast in it?' Mill's *Logic*, Vol. I. p. 212. This is a clever scolding, no doubt; but, as it seems to us, indifferent logic. The phraseology itself is highly objectionable. In order to make out that the conclusion is anticipated in the premisses, though not foreseen by the reasoner, Mr Mill resorts to a doctrine of '*unconscious assertion*' which we can only compare with the hidden sense of prophecy imagined by divines. 'Assertion' not being an automatic articulation by the lips, but a mental act, the intentional predication of a certain attribute present in thought respecting a certain subject also present in thought can not be 'unconscious'; and the epithet does but evade the fact that the assertion in question is not there at all. To another mind, indeed, and to the same mind at a future time, the proposition may suggest the application which the sentence as uttered did not contemplate: but these are phenomena foreign to the immediate act of predication, and not entitled to be imported into its description. And as to Mr Mill's demand that no general proposition shall be uttered till the speaker holds in his thought all instances to which it may be applied, we know of nothing more simply impossible or more entirely destructive of all scientific method whatever. The foresight of its particular cases is *not* 'fairly included' in the meaning or in the evidence of a general rule; and a person may reasonably assent to the law of refraction without any suspicion

of the vast compass of facts over which its interpretation ranges. There are grounds,—whatever account we may give of them,—for ascribing attributes to certain *natures* or *kinds* of being, without going through the objects included under them or having any prescience of their actual contents. It is not necessary to know the natural history of all the varieties of mankind before we can venture to affirm mortality of human beings in general. To revert to our old syllogism: 'All birds are warm-blooded, swallows are birds; therefore swallows are warm-blooded.' It is surely possible (1) to think the attribute 'warm-blood' of the genus (bird) without thinking it of the species (swallow),—that is, to have the *major* premiss without the conclusion; (2) to ascribe to the species (swallow) the nature of the genus (bird) without therewith ascribing to it all the concomitants (as warm-blood) of the genus,—that is, to have the *minor* premiss without the conclusion. But it is *not* possible to do both these things without at once recognizing the conclusion. This is all that is required by the theory of the syllogism; and against this Mr Mill can only urge, that *if* it be true,—why, it *ought not* to be true[1]."

According to Dr Martineau, therefore, the syllogism does not involve the fallacy of *petitio principii*,—(1) because the conclusion is not present in thought while the major or the minor premiss is, and (2) because the conclusion does not follow from the major alone, nor from the minor alone, but from the two taken jointly. The second point is quite self-evident and follows from the definition of a syllogism. The conclusion does not follow from either of the premisses singly, but from both of them taken jointly. Professor De Morgan[2] and a writer in the *British Quarterly Review* (August, 1846) also point out that Mill's view makes the minor proposition of a syllogism quite superfluous and unnecessary, and that as the minor premiss is an essential part of the syllogistic argument, the conclusion not being deducible from the major premiss alone, Mill's objection to the syllogism is

[1] *Essays*, Vol. II. pp. 356—359.
[2] *Formal Logic*, pp. 257—259.

quite untenable. "The whole objection," says De Morgan, "tacitly assumes the superfluity of the minor, that is, tacitly assumes we know Plato to be a man as soon as we know him to be Plato[1]." The reviewer says that if the major premiss included the conclusion, "we should be able to affirm the conclusion without the intervention of the minor premiss; but every one sees that that is impossible." No general proposition can be applied to a new case, unless a minor proposition affirms the new case to come under the general or to possess the marks characteristic of the subject of the general.

In reply to the first point Mill would of course say that though the conclusion is not present in thought, it ought to have been, that no one ought to admit the major without seeing that he thereby also admits the conclusion. Martineau admits that all this is actually seen by the teacher, but that it is not seen by the learner. Hence what may be a *petitio principii* to the former is not so to the latter. The value of an argument depends on the state of the mind to which it is addressed. To the omniscient mind all reasoning must involve a *petitio principii*. To us what is a *petitio principii* at one time was not so at another. If we can *somehow get a general proposition* without actually thinking of, or observing, all the particulars to which it is applicable, then the syllogism can not reasonably be said to be guilty of the charge of a *petitio principii*. "There are," says Martineau, "grounds,—whatever account we may give of them,—for ascribing attributes to certain *natures* or *kinds* of being, without going through the objects included under them or having any prescience of their actual contents." This is the question of questions. Can we ascribe attributes to certain *natures* or *kinds* of being, without having examined *all* the particular objects included under them? In other words, can we establish the truth of a universal proposition from the truth of certain cases included in it, without examining all the possible cases? This is the great problem of Inductive Logic. It is the business of Inductive

[1] *Formal Logic*, p. 259.

Logic to lay down rules and canons, to which we must conform, in order that we may infer general or universal propositions from particular ones. Deductive Logic takes for granted that there *are* universal propositions, whatever account may be given of their origin, nature, and grounds by philosophers of different schools. If there are such propositions, the syllogism can not reasonably be regarded as a *petitio principii;* it becomes, on the contrary, a very useful and sound process of reasoning. If it can be quite satisfactorily established, for example, by the rules and canons of Induction from the observation of *some* cases, that the attribute B is a mark of A,—that wherever B is, A is; and if in a new case C, I find the attribute B, I can reasonably infer the attribute A, of which the former is, by supposition, an unfailing mark. This reasoning, when fully expressed, gives rise to the following syllogism "All B is A, C is B, therefore C is A." It may be also thus stated, "A co-exists with B, B co-exists with C, therefore A co-exists with C[1]." Here, in establishing the major premiss, the *new* case in question was not in any way concerned. It had in fact no existence at all, real or imaginary, and therefore could not be known, or thought of, when the major was established. You may of course doubt the truth of the major premiss, or that the new case in question has the attribute B; but granting both the premisses to be true, you can not doubt the conclusion,—you must regard it as certain. And this brings us to the question of the proper nature of Deductive Inference.

§ 5. *Hypothetically necessary* character of all Deductive Inference. In deductive or syllogistic reasoning we draw conclusions from given propositions as data. Given the premisses, we infer the conclusion that follows *necessarily* from them. We are not in any way concerned to prove our premisses; but our conclusion must be true, if the premisses be true. Hence it is evident that the truth we arrive at by deductive or syllogistic reasoning is entirely of a hypothetical character, depending for

[1] *Vide* Appendix A, pp. 282—284.

its trustworthiness entirely on the trustworthiness of the data. If the latter be true, the former must be so. The premisses of a syllogism, though they may be immediately the conclusions of prior syllogisms, are ultimately the results of Induction, Observation, Perception, or Intuition; but whatever their origin may be, Deductive Logic has nothing to do with it. All that it is concerned with is, the legitimacy of the conclusion or conclusions that are drawn from the premiss or premisses. To its student Deductive Logic offers the following wholesome advice:—"If you wish to live happily in my domain, obey my Laws. If you desire to enjoy the peace of certitude, conform to the rules and conditions I have laid down. I take no account of your prejudices, passions, instincts, habits, associations, interests, and tendencies, which may induce you to infer any thing from any thing else: you must, under all circumstances, implicitly or explicitly obey my Laws, if you desire to attain your object. If you reason from particulars to particulars, you reason against my express Law, and though your conclusions may in some cases be accidentally true, the means you employ to attain your end are none the less unlawful. If you reason from some to all, you do this at your own risk and responsibility. The Law which I lay down is that you infer the particular from the general, or the less general from the more general, and not conversely."

CHAPTER IX.

Probable Reasoning and Probability.

§ 1. If both the premisses of a syllogism are necessary, or assertory, or probable, the conclusion is necessary, assertory, or probable. If the modality of one premiss be different from that of the other, the conclusion has the less certain modality. For example, 'B must be A, C is B: ∴ C is A'; 'B is A, C is probably B: ∴ C is probably A.' Now what is the meaning of the propositions 'C is probably B' and 'C is probably A'? From the two premisses 'A is probably B' and 'B is probably C,' we may infer 'A is probably C.' Is this inference always legitimate? Is the meaning of *probably*, or rather is the *degree of probability*, the same in the conclusion as in either of the premisses? Under what conditions is the conclusion valid? In order to answer these questions, we must first of all state the meaning of a Probable Proposition.

§ 2. The Meaning of a Probable Proposition.

'It will *probably* rain to-morrow,' or 'He will *probably* die,' means, *subjectively*, that my belief in the event in question is not full or complete, is of a degree less than the highest; and *objectively*, that the evidence for the happening of the event in question is not of such a nature as to make it a certainty. That this is the meaning of the proposition will be evident if we consider the meaning in the assertory form. 'It rains,' 'He is dead,' 'The sun rises,' 'Fire is burning': in each of these my belief is of the highest degree, and the event in ques-

tion is quite certain: *subjectively*, there is no trace of doubt, and *objectively*, there is not the least uncertainty about the event. When the word *probably* is added to the copula, the proposition means, *subjectively*, that the state of my mind in regard to the event is a mixture of belief and doubt, partial belief caused by certain evidence for, and partial doubt caused by certain evidence against, the event, that is, a state of incomplete belief caused by incomplete evidence for the event; and it means, *objectively*, that there is some evidence for, and some against, the event, or at any rate that all the evidence attainable is not such as to make the event a certainty. For example, 'He will probably die' means that there are certain appearances that are symptoms of death, and that there are others which are not: that there are certain signs or marks from which we may infer that death will result, and that there are others from which we may infer the contrary; so that altogether the evidence is conflicting, and the state of mind resulting may be said to be a state of partial belief, or a mixture of belief and doubt.

In this sense the words 'probably,' 'probable,' 'probability' mean any degree of belief less than the highest, and any evidence for the event less than certainty. If we represent full belief and highest certainty by 1, we may represent different degrees of 'probability' by fractions such as $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. In ordinary language the word 'probable' means 'more likely than not,' and in this sense 'probability' would always be represented by fractions greater than $\frac{1}{2}$. But, in the widest sense in which it is used here, it may be represented by any fraction however small or large, and corresponds exactly to the mathematical word 'chance.'

The probability of a proposition may, then, be represented by a fraction. But what is the exact meaning of the fraction, and how do we get it? The meaning of the proposition 'It will probably rain to-morrow' is, we may say, that the probability of its raining to-morrow is $\frac{m}{n}$; or the meaning of the proposition 'He will probably die this year' is that the probability

of his dying is $\frac{2}{3}$, or $\frac{1}{3}$, or any other fraction. Now, how is this fraction obtained, and what is its real meaning? We cannot discuss this question here. We shall adopt the view held by Dr Venn, which appears to be the best and most reasonable. "I consider," says he, "that these terms (probability, chance) presuppose a series; within the indefinitely numerous class which composes a series, a smaller class is distinguished by the presence or absence of some attribute or attributes. * * * * These larger and smaller classes respectively are commonly spoken of as instances of the 'event,' and of 'its happening in a given particular way.' Adopting this phraseology, which, with proper explanations, is suitable enough, we may define the probability or chance (the terms are here regarded as synonymous) of the event happening in that particular way as the numerical fraction which represents the proportion between the two different classes in the long run. Thus, for example, let the probability be that of a given infant living to be 80 years of age. The larger series will compose all men, the smaller all who live to 80. Let the proportion of the former to the latter be 9 to 1; in other words, suppose that 1 infant in 10 lives to 80. Then the chance or probability that any given infant will live to 80 is the numerical fraction $\frac{1}{10}$.[1]" Conversely, if the probability of a man living to 80 be $\frac{1}{10}$, this implies that in every 10 persons one only lives to that age. Similarly, if the probability of its raining to-morrow be $\frac{2}{3}$, this implies that in every three cases like the present, rain happens in two cases on the following day. If the probability of a man's dying of a certain disease be $\frac{1}{3}$, it means that in every three cases of that disease one dies. The two classes, one larger and the other smaller, the proportions between which constitute the probability are, in the last example, (1) the class of persons who have had that disease, and (2) the special class within the other of persons who have died of it; and the proportion of the second to the first is represented by the fraction $\frac{1}{3}$.

[1] Venn's *Logic of Chance*, 2nd ed., p. 145.

§ 3. The Rules of Immediate Inference.

Every probable proposition is thus connected with what Dr Venn aptly calls a *Proportional* proposition of the form 'm A's in n are B.' It can be shown that every probable proposition must ultimately be traced to a proportional proposition of that form, and that, without tracing it to such a proposition, we can give no rational account of its meaning, when the probability is represented by a fraction. A *proportional* proposition is to be distinguished from a *universal* of the form 'All A is B.' From the latter we may infer that 'Any A or sub-class of A is B.' From the former we may infer that 'Any A is *probably* B,' the probability being represented by the fraction $\frac{m}{n}$. Given that 9 men in 10 of any assigned age live to 40, we may immediately infer that the probability of a man of that age living to 40 is $\frac{9}{10}$. Given that 3 in 4 men in India are Hindus, we may immediately infer that the probability of a man in India being a Hindu is $\frac{3}{4}$. Given that 2 in 4 candidates will pass at the examination, we may immediately infer that the probability of a candidate's passing is $\frac{1}{2}$. Thus, from every proportional proposition, we may infer a probable one, the probability of which is represented by a fraction. Conversely, from a probable proposition we may infer a proportional one. Given the probable proposition 'A is probably B,' the probability of which is represented by the fraction $\frac{2}{3}$, we may infer the proportional proposition '2 in 3 A's are B.' Given that the probability of a man under certain circumstances becoming rich is $\frac{1}{10}$, we can immediately infer that 1 man in 10 under the same circumstances becomes rich. Given that the probability of an event happening is $\frac{3}{5}$, we can infer that 3 events in 5 of that nature do usually happen.

Examples.

'Most A's are B': from this we can infer that the probability of any A being B is greater than $\frac{1}{2}$.

'$\frac{3}{4}$ of A are B' or '3 A's in 4 are B': from this we can infer that the probability of any A being B is $\frac{3}{4}$.

'Some A's are B': from this we can infer nothing. Similarly from 'Many A's are B' we can infer nothing. The proportion of A that is B must be stated before we can infer the probability of any A being B., *i.e.*, the proposition must be a proportional one.

'The greater number of A are B': from this we can infer that the probability of any A being B is greater than $\frac{1}{2}$.

The same inference follows from the proposition 'the majority of A are B.'

§ 4. The Rules of Mediate Inference.

The rules of mediate inference in probability may be divided into two classes—(1) those which are *formal*, and (2) those which are more or less *experimental*. The former follow necessarily from probable propositions by the mere application of arithmetic; whilst the latter "either depend upon peculiar hypotheses, or demand for their establishment continually renewed appeals to experience and extension by the aid of the various resources of Induction."

§ 5. (1) The Formal Rules of Mediate Inference.

"The fundamental rules of probability strictly so called, that is, the *formal* rules, may be divided into two sub-classes—(i) those obtained by addition or subtraction on the one hand corresponding to what are generally termed the connection of *exclusive or incompatible* events, and (ii) those obtained by multiplication or division on the other hand corresponding to what are commonly termed *dependent* events[1]."

(i) Rules of *Exclusive* Events.—"If the chances of two exclusive or incompatible events be respectively $\frac{1}{m}$ and $\frac{1}{n}$, the chance of one or other of them happening will be $\frac{1}{m}+\frac{1}{n}=\frac{m+n}{mn}$. Similarly, if there were more than two events of the kind in question." A bag, for example, contains 16 balls, of which 10 are red and 6 are blue. That is, 10 balls in 16 being red, and 6 in 16 being blue, the chances of drawing a red and a blue ball

[1] Venn's *Logic of Chance*, 2nd ed., p. 150.

are respectively $\frac{10}{16}$ and $\frac{6}{16}$. Therefore the chance of drawing either is the sum of $\frac{10}{16}+\frac{6}{16}=1$; that is, the ball drawn is certain to be a red or a blue ball and can not be anything else. The events here are exclusive or incompatible, because while one happens the other can not; when a red ball, for instance, is drawn, a blue ball can not be drawn at the same time. I may of course draw two balls one after another, but, while drawing once, one ball must be drawn, and it must be either red or blue. Suppose the ball first drawn is a red one, and is not replaced in the bag; then there are now 9 red and 6 blue balls in the bag, and the chances respectively are $\frac{9}{15}$ and $\frac{6}{15}$. Suppose at the second drawing a blue ball is drawn; now there are 9 red and 5 blue balls in the bag, and the chances respectively are $\frac{9}{14}$ and $\frac{5}{14}$. Suppose on drawing a third time a red ball comes out; now there are in the bag 8 red and 5 blue balls, and the chances respectively are $\frac{8}{13}$ and $\frac{5}{13}$.

The following is a sort of corollary to the above:—

"If the chance of one or other of two incompatible events be $\frac{1}{m}$ and the chance of one alone be $\frac{1}{n}$, the chance of the remaining one will be $\frac{1}{m}-\frac{1}{n}=\frac{n-m}{mn}$. For example, if the chance of any one dying in a year is $\frac{1}{10}$, and his chance of dying of some particular disease is $\frac{1}{100}$, his chance of dying of any other disease is $\frac{9}{100}$.[1]" In the example given above, the chance of drawing a red or a blue ball is 1, and the chance of drawing a blue ball is $\frac{6}{16}$, therefore the chance of drawing a red ball is $1-\frac{6}{16}=\frac{10}{16}=\frac{5}{8}$.

(ii) Rules of *Dependent* Events.—"We can also make inferences by multiplication or division. Suppose that the two events instead of being incompatible are connected together in the sense that one is contingent upon the occurrence of the other. Let us be told that a given proportion of the members of the class or series possess a certain property, and a given proportion again of these possess another property, then the proportion of the whole

[1] Venn's *Logic of Chance*, p. 152.

which possess both properties will be found by multiplying together the two fractions which represent the above two proportions. Of the inhabitants of London, twenty-five in a thousand, say, will die in the course of the year; we suppose it to be known also that one death in five is due to fever; we should then infer that one in 200 of the inhabitants will die of fever in the course of the year. It would, of course, be equally simple by division to make a sort of converse inference. Given the total mortality *per cent.* of the population from fever, and the proportion of fever cases to the aggregate of other cases of mortality, we might have inferred, by dividing one fraction by the other, what was the total mortality *per cent.* from all cases.

"The rule, as given above, is variously expressed in the language of probability. Perhaps the simplest and best statement is that it gives us the rule of dependent events, that is, if the chance of one event is $\frac{1}{m}$, and the chance that if it happens another will also happen is $\frac{1}{n}$, then the chance of the latter is $\frac{1}{mn}$. In this case it is assumed that the latter is so entirely dependent upon the former that, though it does not always happen with it, it certainly will not happen without it; the necessity of this assumption, however, may be obviated by saying that what we are speaking of in the latter case is the *joint* event, viz., both together if they are simultaneous events, or the latter *in consequence of* the former, if they are successive[1]."

Examples of (ii).

Suppose the chance of a boy of 10 years living to 20 is $\frac{2}{3}$, and if he lives to that age the chance of his being educated is $\frac{1}{3}$, then the chance of his being educated is $\frac{2}{3}\times\frac{1}{3}=\frac{2}{9}$. That is, 2 boys in 9 of the age of 10 years live to 20 and become educated.

Suppose the chance of there being plenty of rain this season is $\frac{3}{4}$, and the chance of the crops growing $\frac{4}{5}$, if the former event happens;

[1] Venn's *Logic of Chance*, pp. 153—4.

then the chance of there being rain and of the crops growing, *i. e.*, of the *joint* event, is $\frac{3}{4} \times \frac{5}{6} = \frac{5}{8}$. This is in fact the chance of the last event, which happens *in consequence* of the first.

Suppose the chance *of* a person's acting prudently under certain circumstances is $\frac{2}{3}$, and if he acts prudently, the chance of his being happy is $\frac{5}{6}$, then the chance of his both acting prudently and being happy, or, in other words, of his being happy which happens *in consequence of* the first event, is $\frac{2}{3} \times \frac{5}{6} = \frac{10}{18} = \frac{5}{9}$. As the second event depends in all these cases upon the first, as the happening of the one is dependent upon the happening of the other, the two events are called *Dependent* or *Contingent* events, and should be distinguished, on the one hand, from incompatible events, and, on the other, from independent events.

Similarly, if the chance of A being B is $\frac{3}{4}$, and if, this happening, the chance of B being C is $\frac{1}{3}$, then the chance of A being C is $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$. That is, 1 A in 4 is C.

Here we may take up the example given at the beginning of this chapter, and state the condition under which the inference is valid so far as we are able to do at present:—

$$\text{A is probably B} \left(\text{probability} = \frac{1}{m}\right),$$

$$\text{B is probably C} \left(\quad ,, \quad = \frac{1}{n}\right);$$

$$\therefore \text{A is probably C} \left(\quad ,, \quad = \frac{1}{mn}\right).$$

Here the probability of the conclusion will be the product of the probabilities of the premisses, if they are dependent events in the sense explained above. That is, the reasoning will be valid if it admits of being stated as follows:—"The chance of A being B is $\frac{1}{m}$, and, this happening, the chance of B being C is $\frac{1}{n}$, then the chance of A being C is $\frac{1}{m} \times \frac{1}{n} = \frac{1}{mn}$"; or as follows: "One A in m is B, and, this happening, one B in n is C, therefore one A in m × n is C"; or as follows: "A is probably B

$\left(\text{probability}=\frac{1}{m}\right)$, whatever A is B is probably C $\left(\text{probability}=\frac{1}{n}\right)$; therefore A is probably C $\left(\text{probability}=\frac{1}{mn}\right)$." These three statements express in different ways the same matter of fact.

§ 6. (2) The *Experimental* Rules of Mediate Inference.

The rules of this class "stand upon a somewhat different footing from the above in respect of their cogency and freedom from appeal to experience or to hypothesis. In the first class, we considered cases in which the data were supposed to be given under the condition that the propositions which distinguish the different kinds of events, whose frequency was discussed, were respectively known to be disconnected and known to be connected. Let us now suppose that no such conditions are given to us. One man in 10, say, has black hair, and 1 in 12 is short-sighted, what conclusion could we then draw as to the chance of any given man having one only of these two attributes, or neither, or both? It is clearly possible that the properties in question might be inconsistent with one another, so as never to be found combined in the same person, or all the short-eyed might have black hair, or the properties might be allotted in almost any other proportion whatever, except as restricted by the arithmetical conditions. If we are perfectly ignorant upon these points, it would seem that no inferences whatever could be drawn about the required chances[1]." If, on the other hand, we are warranted in making the assumption that "the division into classes caused by each of the above distinctions should subdivide each of the classes created by the other distinction in the same ratio in which it subdivides the whole," then the following rule of inference will hold good:—

"If the chances of a thing being p and q are respectively $\frac{1}{m}$ and $\frac{1}{n}$, then the chance of its being both p and q is $\frac{1}{mn}$, the

[1] Venn's *Logic of Chance*, pp. 154—5.

chance of its being p and not q is $\frac{n-1}{mn}$, and the chance of its being not p and not q is $\frac{(m-1)(n-1)}{mn}$, where p and q are independent. The sum of these chances is obviously unity, as it ought to be, since one or other of the four alternatives must necessarily exist." This is the rule of the so-called *independent* events, the nature of the independence being defined by the supposition stated above.

Taking the instance mentioned above, "let us take a batch of 1,200 as a sample of the whole. Now, from the data which were originally given to us, it will easily be seen that in every such batch there will be on the average 120 who have black hair, and therefore 1,080 who have not. And here in strict right we ought to stop, at least until we have appealed again to experience; but we do not stop here. From data which we assume," that is, from the data which follow from granting the assumption stated above to be true, "we go on to infer that of the 120, 10 (*i.e.* $\frac{1}{12}$ of 120) will be short-sighted, and 110 (the remainder) will not. Similarly we infer that of the 1,080 90 are short-sighted, and 990 are not. On the whole, then, the 1,200 are thus divided:—Black-haired, short-sighted, 10; short-sighted without black hair, 90; black-haired men who are not short-sighted, 110; men who are neither short-sighted nor black-haired, 990." If that assumption had not been true, we should not have been justified in drawing those inferences, for of the 1,200 there would be 120 black-haired; and the 100 short-sighted might be none of the 120 who had black hair, and so forth. The necessary and sufficient condition of our inferences being valid is that of the 120 who have black hair, 10 must be short-sighted, as also the same proportion of the 1,080 who have not black hair; and that taking likewise the short-sighted first, of the 100 who are short-sighted, 10 must have black hair as well as the same proportion of the 1,100 who are not short-sighted. That is, the condition which is assumed to be true is, that the division into classes caused by each of the given distinctions should subdivide each of

the classes created by the other distinctions in the same ratio in which it subdivides the whole. This condition being true, the rule of inference given above is quite correct and free from all objection. In the form in which it is usually given it is open to objection, and leads to inferences which are not formally valid, the events being assumed to be independent where nothing is known about the distribution of the properties. But we have seen that it is necessary that we should possess some positive knowledge of the distribution before we can apply the rule.

We can now further state the *condition* or *supposition* under which the inference is valid in the case of our original example :—

$$\text{A is probably B} \left(\text{probability} = \frac{1}{m}\right),$$
$$\text{B is probably C} \left(\quad \text{,,} \quad = \frac{1}{n}\right);$$
$$\therefore \text{A is probably C} \left(\quad \text{,,} \quad = \frac{1}{mn}\right).$$

The probability of the conclusion will be the product of the probabilities of the two premisses, *if the subdivision of A created by B be subdivided in the same ratio in which the whole of B is subdivided by C.* For example, suppose the probabilities to be $\frac{2}{3}$ and $\frac{1}{3}$ respectively, and A to be represented by a sample of 36; then, according to the first premiss, 24 A's in 36 are B, and according to the second premiss, and the condition assumed, 8 B's in these 24 are C: therefore, 8 A's in 36 are C,—that is, the probability of A being C is $\frac{8}{36}$, or $\frac{2}{9}$, which is equal to the product of the probabilities of the two premisses.

In certain cases, however, it is possible to draw valid inferences without any assumption whatever—I mean those cases in which the sum of the probabilities of the two *independent* events exceeds unity, and in which the two premisses are as in the third syllogistic figure. The rule of inference in such cases is as follows:—if the chances of a thing being p and q are, respectively, $\frac{1}{m}$ and $\frac{1}{n}$, then the chance of its being both p and q

is $\frac{1}{m}+\frac{1}{n}-1$, and the chance of its being p and not-q is $\frac{1}{m}-\frac{1}{n}$, if $\frac{1}{m}$ be greater than $\frac{1}{n}$, where p and q are independent. For example, 3 A's in 4 are B, and 1 A in 3 is C; therefore 1 A in 12 must be both B and C,—that is, the probability of A being B is $\frac{3}{4}$, and the probability of A being C is $\frac{1}{3}$, therefore the probability, according to the given rule, of A being both B and C is $\frac{3}{4}+\frac{1}{3}-1$, or $\frac{1}{12}$. Let A be 24 men, of whom $\frac{3}{4}$, that is 18, are educated, B, and $\frac{1}{3}$, that is 8, are rich, C, then $\frac{3}{4}+\frac{1}{3}-1$, or $\frac{1}{12}$, that is 2, must be both educated and rich, and $\frac{3}{4}-\frac{1}{3}$, or $\frac{5}{12}$, that is 10, both educated and not rich. Similarly, if $\frac{4}{5}$ of A are B and if $\frac{3}{5}$ of A are C, then $\frac{2}{5}$ of A must be both B and C, and $\frac{1}{5}$ of A both B and not C. From the first conclusion it follows that some B's are C, and some C's are B.

§ 7. Exercises.

1. Fully explain the meaning of the following propositions, and draw the inferences which follow from them:—

(*a*) The substance A is probably a metal.
(*b*) B is probably a prudent man.
(*c*) D will probably pass at the F. A. Examination.
(*d*) E will probably live to the age of eighty.
(*e*) The sun will most probably rise to-morrow.
(*f*) All virtuous men are probably happy.
(*g*) This fossil is probably carboniferous.
(*h*) The luminiferous ether probably gravitates.

2. The probability of a fossil being mesozoic is $\frac{1}{2}$; and if it is mesozoic, the probability of its being cretaceous is $\frac{3}{4}$; and if it is cretaceous, the probability of its being found in the English chalk formation is $\frac{7}{8}$. Calculate the probability of the fossil being found in the English chalk.

3. The probability of a new-born child living to the age of 25 years is $\frac{4}{5}$; and if it lives to that age, the probability of its being well-educated is $\frac{1}{4}$; and if it is well-educated, the probability of its being a distinguished person is $\frac{1}{20}$. Calculate the probability of the new-born child being a distinguished person.

4. Three elements in four are conductors of electricity; four in five are conductors of heat: draw the conclusions which follow if the total number of elements be sixty.

5. The population of London is 4,000,000; and there are 500 distinguished men in all professions, suppose 100 in the medical, 150 in the literary, 100 in the army and navy, 50 in the church, 25 great orators, 25 engineers, 50 scientific men; what is the probability of a Londoner being a distinguished man, and belonging to any of these professions?

6. Test the following:—(1) Most A's are B; most B's are C: therefore some A is C. (2) Most A's are B; most A's are C: therefore some B is C.

7. The probability of a man's dying within two years of fever is $\frac{1}{8}$, of cholera $\frac{2}{3}$, of consumption $\frac{1}{4}$. What are the probabilities of his dying of fever or consumption, of cholera or consumption, and of fever or cholera?

8. Thirty *per cent.* of fishes are edible, and twenty *per cent.* are freshwater. Calculate the probabilities of a fish being edible, freshwater, edible and freshwater, edible and not-freshwater, freshwater and not-edible, and not-edible and not-freshwater.

APPENDIX.

A.—Canons or Axioms of the Syllogism according to Logicians.

§ 1. *Lambert's Canons for the so-called Imperfect Figures.*—In opposition to the view that all the figures except the first are imperfect, because they have no canons of their own like the '*Dictum de Omni et Nullo*' for the first or *perfect* figure, and that, therefore, syllogisms in those figures must be reduced to the first, Lambert (in his *Neues Organon*, Leipzig, 1764) enunciates a distinct canon for each figure, and thus places them all on an equality. For the first figure Lambert recognizes the 'dictum de omni et nullo' as usual. For the second figure he lays down a canon called '*Dictum de Diverso*,' which is as follows:—"If one term be contained in, and another excluded from, a third term, they are mutually excluded." This dictum is as self-evident as the 'dictum de omni et nullo.' On applying it to the sixteen possible combinations of premisses it will be found that the same valid moods are obtained as on any other method. It holds good in the moods *Cesare, Camestres, Festino, and Baroko.* In *Cesare* the term 'C' (taking A, B, and C as standing for the major, middle, and minor terms respectively) is included in 'B' in the minor premiss, and in the major premiss the term 'A' is excluded from 'B'; therefore, according to the 'dictum de diverso,' 'C' and 'A' are excluded from each other, that is, the conclusion is 'No C is A.' In *Baroko* the term 'A' is included

in 'B' in the major premiss, and the term 'some C' is excluded from 'B' in the minor premiss; therefore, according to the same dictum, 'Some C' and 'A' are excluded from each other, that is, the conclusion is 'Some C is not A.' The 'dictum de diverso' is similarly applicable to *Camestres* and *Festino*, and thus distinguishes the valid from the invalid moods in the second figure.

For the third figure Lambert enunciates the following canon, which is called '*Dictum de Exemplo*':—"Two terms which contain a common part, partly agree, or if one term contains a part which the other does not, they partly differ." This is also self-evident, and may be easily applied to syllogisms in the third figure. In the valid mood *Darapti* of this figure 'B' is a part of 'A' in the major premiss, and also a part of 'C' in the minor premiss, that is, 'A' and 'C' have a common part 'B'; therefore they partly agree, that is, 'Some C is A,' according to the first part of the 'dictum de exemplo.' In the mood *Felapton* of the same figure the term 'C' contains 'B' in the minor premiss, while 'B' is not contained in 'A,' according to the major premiss; therefore 'C' and 'A' partly differ, that is, 'Some C is not A,' according to the second part of the same dictum. The first part of the 'dictum de exemplo' is similarly applicable to the other affirmative moods, and the second part to the other negative moods; and thus it distinguishes the valid from the invalid moods in the third figure.

For the fourth figure Lambert gives a canon called '*Dictum de Reciproco*,' which is stated as follows[1]:—"If no M is B, no B is this or that M; if C is or is not this or that B, there are B's which are or are not C." But it may be more clearly stated thus: If a term be included in a second term which is excluded from a third, then the third is excluded from the first; if a term be included in (or excluded from) a second term which is included in a third, then a part of the third is included in (or

[1] *Vide* Mansel's *Aldrich* (1849), p. 80; Hamilton's *Lectures*, Vol. IV. p. 441; and Ueberweg's *Logic*, p. 372.

excluded from) the first. The first part is applicable to the mood *Camenes*, while the second part is applicable to the moods *Bramantip*, *Dimaris*, *Fesapo*, and *Fresison* in the fourth figure. Both parts of the dictum are self-evident, and require no explanation.

Lambert not only abolishes Reduction, and gives a canon for each of the so-called imperfect figures, but he also establishes their independence of the first figure and their equality with it, by showing that each figure is by its nature especially adapted for a particular kind of argument, and that we naturally think and express our thoughts in certain cases in one figure rather than in another. "For example, the proposition, *Some stones attract iron*, everyone will admit, because *The magnet is a stone* and *attracts iron*. This syllogism is in the third figure. In the first, by conversion of one of its premisses, it would run thus :—

(A)		All magnets attract iron	...	(major premiss),
(I)		Some stones are magnets	...	(minor premiss);
(I)	∴	Some stones attract iron	...	(conclusion).

"Here we are unaccustomed to the minor proposition, while it appears as if we must have all stones under review, in order to pick out magnets from among them. On the other hand, that *the magnet is a stone* is a proposition which far more naturally suggests itself, and demands no consideration. In like manner:—*A circle is no square;—for the circle is round,—the square not.* This proof (in the second figure) is as follows, when cast in the first :—

What is not round is no circle,
A square is not round,
Consequently, &c.

"Here the major proposition is converted by means of a *terminus infinitus* (*i.e.* contraposed), and its truth is manifested to us only through the consciousness that *all circles are round.* For, independently of this proposition, should we not hesitate,—there being innumerable things which are *not* round,—whether the circle were one of those which belonged to this category?

We think not; because we are aware. It is thus apparent that we use every syllogistic figure there, where the propositions, as each figure requires them, are more familiar and more current. The difference of the figures rests, therefore, not only on their form, but extends itself, by relation to their employment, also to things themselves, so that we use each figure where its use is more natural: *The First for finding out or proving the Attributes of a thing: the Second for finding out or proving the Difference of things; the Third for finding out and proving Examples and Exceptions; the Fourth for finding out and excluding Species of a Genus*[1]."

Mill has the following lines on Lambert and his work: "A German philosopher, Lambert, whose *Neues Organon* (published in the year 1764) contains among other things one of the most elaborate and complete expositions which have ever been made of the syllogistic doctrine, has expressly examined which sorts of arguments fall most naturally and suitably into each of the four figures; and his investigation is characterized by great ingenuity and clearness of thought. His conclusions are: 'The first figure is suited to the discovery or proof of the *properties* of a thing; the second to the discovery and proof of the *distinction* between things; the third to the discovery or proof of *instances* and *exceptions;* the fourth to the *discovery* or *exclusion* of the different species of a genus.' The reference of syllogism in the last three figures to the 'dictum de omni et nullo' is, in Lambert's opinion, strained and unnatural; to each of the three belongs, according to him, *a separate axiom, co-ordinate and of equal authority*, with that dictum, and to which he gives the names of 'dictum de diverso' for the 2nd figure, 'dictum de exemplo' for the 3rd, and 'dictum de reciproco' for the 4th. . . ' . . . . . Mr Bailey (*Theory of Reasoning*, 2nd edition, pp. 70—74) takes a similar view of the subject[2]." A similar view is also taken by Archbishop Thomson and by Dr Martineau.

[1] Hamilton's *Lectures*, Vol. IV. p. 439.

[2] Mill's *Logic*, Vol. I. pp. 194—5.

§ 2. *Thomson's Canons.*—Thomson regards the following law as the general canon upon which all mediate inference depends:—"The agreement or disagreement of one conception with another is ascertained by a third conception, inasmuch as this wholly or by the same part agrees with both, or with only one, of the conceptions to be compared[1]."

For the first figure he modifies it thus:—"The agreement or disagreement of a subject and predicate is ascertained by a third conception, predicate to the former and subject to the latter; inasmuch as this wholly or by the same part agrees with both, or with one only, of the conceptions to be compared[2]."

For the second figure he modifies it thus:—"The agreement of two conceptions is ascertained by a third conception, which stands as predicate to both; inasmuch as this wholly or by the same part agrees with both, or with one only, of the conceptions to be compared[3]."

For the third figure he modifies it thus:—"The agreement of two conceptions is ascertained by a third conception, which stands as subject to both; inasmuch as this wholly or by the same part agrees with both, or with one only, of the conceptions to be compared[3]."

Thomson recognizes only three figures, and dismisses the fourth, on the ground that, in the conclusion in that figure, what was the predicate in a premiss becomes the subject, and what was the leading subject in a premiss becomes the predicate. This, he says, is not the natural order, but that order wholly inverted. The natural order is seen in the first, somewhat distorted in the second and third, and *wholly inverted* in the fourth, against which the mind rebels[3]. These special canons, as well as the general law, are quite self-evident, and do not require any explanation. They are directly applicable to the syllogism in each figure, and make Reduction unnecessary and superfluous.

[1] Thomson's *Laws of Thought* (1864), p. 163.
[2] *Ibid.* p. 175. [3] *Ibid.* pp. 177—8.

§ 3. *Whately's Canons.*—Whately regards the '*dictum de omni et nullo*' as the *ultimately* supreme Rule or Maxim of all reasoning; but as this is not *directly* applicable to all syllogisms, he gives the following two canons for all pure categorical syllogisms:—(1) "If two terms agree with one and the same third, they agree with each other; (2) if one term agrees and another disagrees with one and the same third, these two disagree with each other[1]." The first is for affirmative conclusions, and the second for negative. "On these two canons are built the syllogistic rules or cautions which are to be observed with respect to syllogisms, for the purpose of ascertaining whether those Canons have been strictly observed or not[2]." By these rules Whately determines the valid syllogisms in each figure, and then further confirms those in the 2nd, 3rd, and 4th figures by Reduction to the 1st, to which the 'dictum de omni et nullo' is directly applicable.

§ 4. *Hamilton's Canons.*—Hamilton divides all categorical syllogisms into Deductive and Inductive. The former are divided again into Intensive or Extensive according as the reasoning is in the quantity of comprehension or of extension. All *extensive* syllogisms are regulated by the canon "What belongs to the genus belongs to the species and individual; what is repugnant to the genus is repugnant to the species and individual, or more briefly, what pertains to the higher class pertains also to the lower[3]."

He then gives the following three proximate rules by which a *regularly and fully* expressed extensive categorical syllogism is governed:—(1) "It must have three and only three terms constituting three and only three propositions; (2) of the premisses, the sumption or major premiss must in quantity be definite, that is, universal, and the subsumption or minor premiss in quantity affirmative; (3) the conclusion must correspond in quantity with the subsumption, and in quality with the sumption[4]."

[1] Whately's *Elements*, 9th edn., p. 54.

[2] *Ibid.* p. 54.

[3] Hamilton's *Lectures*, Vol. III. p. 303.

[4] *Ibid.* p. 305.

According to Hamilton syllogisms in the first figure only are fully and regularly expressed, while all syllogisms in the 2nd, 3rd, and 4th figures are irregularly and imperfectly expressed. To the former the three rules are, therefore, directly applicable, while the latter must be regularly and fully expressed, or, in other words, reduced to the first figure, before the rules will be applicable to them. He, however, gives special rules for the 2nd, 3rd, and 4th figures. These rules are the same as those we have given in Part III. ch. III.

All *intensive* syllogisms are regulated by the canon "What belongs to the predicate belongs also to the subject; what is repugnant to the predicate is repugnant to the subject[1]."

In his later writings Hamilton adopts the doctrine of the quantification of the predicate, abolishes the fourth figure, divides the categorical syllogisms into (1) unfigured and (2) figured, and gives the following canons:—

I. "For the unfigured syllogism, or that in which the terms compared do not stand to each other in the reciprocal relation of subject and predicate, being in the same proposition, either both subjects or (possibly) both predicates, the canon is: In so far as two notions (notions proper, or individuals), either both agree, or one agreeing, the other does not, with a common third notion; in so far, these two notions do or do not agree with each other."

II. "For the figured syllogism, in which the terms compared are severally subject and predicate, consequently in reference to each other, containing and contained in the counter wholes of Intension and Extension, the canon is: What worse relation of subject and predicate subsists between either of two terms and a common third term, with which one, at least, is positively related, that relation subsists between the two terms themselves[2]."

Hamilton then gives a canon for each of the three figures. As examples of the unfigured syllogism he gives the following:—

[1] Hamilton's *Lectures*, p. 303.

[2] *Ibid*, Vol. IV. p. 357, and *Discussions*, pp. 653—5.

1. All C and some B are convertible,
   All B and all A are convertible;
   ∴ All C and some A are convertible.

2. A and B are equal,
   B and C are equal;
   ∴ A and C are equal.

§ 5. *Martineau's Canons*[1].—In the chapter on the Theory of Predication we have seen that Dr Martineau holds, for a certain class of propositions, the view, according to which the meaning of a proposition is that the attribute connoted by the predicate belongs to the substance or substances denoted by the subject. Consistently with this view, Dr Martineau would give the following axioms for the first three figures.

For the first figure, in its affirmative relations, the dictum would, according to him, appear in some such form as this:—"Where the same nature both has an attribute and is one, the attribute it has belongs to the substance in which it is." Thus in the mood *Barbara* the same nature B has an attribute A in the major premiss, and is itself one in the minor premiss, being its predicate; therefore the attribute A which B has, belongs to the substance C in which it (B) is, *i.e.* 'All C is A.'

For the second figure the dictum would be as follows:—"If the attribute be present with one nature and absent from another, neither of these can be the attribute of the other."

For the third figure it would be as follows:—"Where two attributes are co-present in the same sphere, each is an attribute of some thing having the other."

The true meaning, according to Dr Martineau, of the syllogism "All birds are warm-blooded, all swallows are birds, therefore all swallows are warm-blooded," is that in the major premiss the subject (birds) is wanted in its denotation, in the minor the same word 'birds' is wanted in its connotation, and in the conclusion the subject 'swallows' is wanted in its denotation and the predicate 'warm-blooded' in its connotation.

[1] *Essays*, Vol. II., 'Theory of Reasoning,' p. 352.

§ 6. *Mill's Canons.*—Mill gives the following two canons or fundamental principles of Syllogism or Ratiocination:—

(1) "A thing which co-exists with another thing, which other co-exists with a third thing, also co-exists with that third thing[1]."

(2) "A thing which co-exists with another thing, with which other a third thing does not co-exist, is not co-existent with that third thing[2]."

"The co-existence meant is," says Mill, "that of being jointly attributes of the same subject. The attribute of being born without teeth, and the attribute of having thirty teeth in mature age, are, in this sense, co-existent, both being attributes of man, though *ex vi termini* never of the same man at the same time[3]."

The first is the principle of affirmative syllogisms, and the second of negative syllogisms. Mill thus analyses an affirmative syllogism:—"All men are mortal, all kings are men; ∴ all kings are mortal. The minor premiss asserts that the attributes denoted by kingship only exist in conjunction with those signified by the word man. The major asserts that the last-mentioned attributes are never found without the attribute of mortality. The conclusion is, that wherever the attributes of kingship are found, that of mortality is found also[4]."

"If the major premiss," continues Mill, "were negative, as 'No men are omnipotent,' it would assert not that the attributes connoted by 'man' never exist *without*, but that they never exist *with* those connoted by 'omnipotent': from which, together with the minor premiss, it is concluded, that the same incompatibility exists between the attribute omnipotence and those constituting a king[4]." That is, the analysis of a negative syllogism, when fully stated, would be as follows:—No men are omnipotent, all kings are men; ∴ no kings are omnipotent. The minor premiss asserts that the attributes of kingship exist only in conjunction with those signified by 'man.' The major asserts that the last-

[1] Mill's *Logic*, Vol. I. p. 203. [2] *Ibid.* p. 204. [3] *Ibid.* p. 205.
[4] *Ibid.* p. 203.

named attributes never exist with those connoted by 'omnipotent.' The conclusion is, that the attributes of kingship never exist with those connoted by 'omnipotent,' or that wherever the former are found, the latter are not found.

For practical purposes, Mill gives the two canons quoted above in a different form founded upon the practical mode of expressing the meaning of a proposition. The real meaning of a proposition like 'All men are mortal' is that the attribute connoted by 'man' exists only in conjunction with the attribute connoted by 'mortal'; that wherever humanity is found, mortality is also found,—that is, the presence of the attribute 'humanity' is a sign or mark of the presence of the attribute 'mortality.' Hence the meaning of an affirmative proposition may, for practical purposes, be taken to be this, that 'the attribute connoted by the subject is a mark of the attribute connoted by the predicate'; and the meaning of a negative proposition, that 'the attribute connoted by the subject is a mark of the absence of the attribute connoted by the predicate.' For example, the proposition 'No men are perfect' means that the attribute 'humanity' is a mark of the absence of 'perfection.' In accordance with this mode of expressing the meaning of propositions, Mill gives the following two axioms or canons for practical purposes:—

(1) "Whatever has any mark has that which it is a mark of," when the minor premiss is a singular proposition with a proper name for its subject.

(2) "Whatever is a mark of any mark is a mark of that which this last is a mark of," when the minor premiss as well as the major is universal.

For example:—If the attribute A is a mark of the attribute B, and if an object has the attribute A, it has also the attribute B,—that is, an object that has the mark (A) has that (B) of which it (A) is a mark. Thus the meaning of the first syllogism, given above, would be as follows:—The objects 'kings' have the mark 'humanity,' which is a mark of 'mortality,' therefore the objects (kings) have the mark 'mortality;' or taking the term

'kings' also in its connotation, the attributes of a king which are a mark of humanity which is a mark of mortality are a mark of the last (mortality). The meaning of the second syllogism given above would be thus expressed:—The attributes of a king, which are a mark of the attributes of humanity, which are a mark of the absence of omnipotence, are a mark of the last (absence of omnipotence).

On this view the general formula of a syllogism is as follows:—

Attribute B is a mark of attribute A,
Attribute C is a mark of attribute B;
∴ Attribute C is a mark of attribute A.

Here B corresponds to the middle term, and A and C to the two extremes, the major and the minor terms. The first statement must be true *in all cases*, and the second *in all or in some cases*, and the conclusion accordingly in *all or in some cases*.

Barbara and *Darii* are thus expressed:

1. In all cases B is a mark of A,
In all (or in some) cases C is a mark of B;
∴ In all (or in some) cases C is a mark of A.

Celarent and *Ferio*, thus:

2. In all cases B is a mark of the absence of A,
In all cases (or in some cases) C is a mark of B;
∴ In all cases (or in some cases) C is a mark of the absence of A.

Mill gives canons for the first figure only, as the other figures can easily be reduced to that, and considers "the two elementary forms of the first figure as the universal types of all correct ratiocination,—the one when the conclusion to be proved is affirmative, and the other when it is negative, even though certain arguments may have a tendency to clothe themselves in the form of the 2nd, 3rd, and 4th figures; which, however, cannot possibly happen with the only class of arguments which are of first-rate importance, those in which the conclusion is an universal affirmative, such conclusions being susceptible of proof in the first figure alone."

B.—The Dilemma according to Logicians.

Whately[1] defines the true Dilemma as "a conditional syllogism with several antecedents in the major and a disjunctive minor."

Mansel[2] defines the Dilemma as "a syllogism, having a conditional major premiss, with more than one antecedent and a disjunctive minor."

Both Whately and Mansel give the following forms:—

I. Simple Constructive—

If A is B, C is D; and if E is F, C is D,
But either A is B, or E is F;
∴ C is D.

II. Complex Constructive—

If A is B, C is D; and if E is F, G is H,
But either A is B, or E is F;
∴ Either C is D, or G is H.

III. Destructive (always complex)—

If A is B, C is D; and if E is F, G is H,
But either C is not D, or G is not H;
∴ Either A is not B, or E is not F.

Whately *excludes* the following forms among others on the ground that they "hardly differ from simple conditional (that is Hypothetical-categorical) Syllogisms":—

(1) If A is B, C is D, E is F, and G is H,
But neither C is D, nor E is F, nor G is H,
∴ A is not B.

(2) If A is B, C is D,
If A is E, G is H,
But neither C is D, nor G is H,
∴ A is neither B nor E.

[1] *Elements*, p. 72. [2] Mansel's *Aldrich*, 1849, p. 93.

(3) If A is B, C is D and also E is F,
But either C is not D, or E is not F,
∴ A is not B.

"The Dilemma is sometimes exhibited," says Mansel, "in another form as a conditional syllogism in which the consequent of the major premiss is disjunctive, and *the whole* denied in the minor,—*e.g.* 'If A is B, either C is D, or E is F, or G is H; but neither C is D, nor E is F, nor G is H; therefore A is not B.' This form is given by Wallis[1] as well as by Wolf and Kant. But it is a perversion of the Dilemma proper, and introduces no distinction whatever, being merely a common disjunctive syllogism, as is shown by Wallis himself."

Professor Fowler[2] defines the Dilemma as "a complex syllogism of which one premiss is a conjunctive (hypothetical), and the other a disjunctive proposition." He follows in the main Mansel and Whately, differing from them only in one point, namely, that the antecedent of the conjunctive premiss may be single as well as double. Thus:—

If A is B, C is D and E is F,
But either C is not D, or E is not F;
∴ A is not B.

Here the antecedent is single.

Three other forms given by Professor Fowler are the same as those given by Mansel and Whately.

Professor Jevons follows Whately and Mansel, and adopts all their forms.

Thomson[3] defines the Dilemma as "a syllogism with a conditional premiss, in which either the antecedent or consequent is disjunctive." He gives the following forms of it:

(1) If A is B or E is F, then C is D,
But either A is B or E is F;
∴ C is D.

[1] Wallis's Lib., III. cap. 19.
[2] *Deductive Logic*, 6th ed., pp. 116—119.
[3] Thomson's *Laws of Thought*, pp. 203—5.

(2) If A is B, then C is D or E is F,
But neither C is D nor E is F;
∴ A is not B.

(3) If some A is B, either the M that are *A*, or the N that are *A*, are *B*,
But neither the M that are A, nor the N that are A, are B;
∴ A is not B.

Hamilton[1]:—"If the sumption (*i.e.* the major premiss) of a syllogism be at once hypothetical and disjunctive, and if in the subsumption (minor premiss) the whole disjunction, as a consequent, be sublated (*i.e.* denied), in order to sublate the antecedent in the conclusion; such a reasoning is called an *Hypothetico-disjunctive syllogism*, or a *Dilemma*. The form of this syllogism is the following:—

"If A exist, then either B or C exists;
But neither B nor C exists;
∴ A does not exist."

"In the sifting of a proposed dilemma, the following points should be carefully examined:—(1) Whether a veritable consequence subsists between the antecedent and consequent of the sumption; (2) whether the opposition in the consequent is thorough-going and valid; and (3) whether in the subsumption the disjunctive members are legitimately sublated. For the example of a dilemma which violates these conditions, take the following:—

If virtue were a habit worth acquiring, it must insure either power, or wealth, or honour or pleasure;
But virtue insures none of these;
Therefore, virtue is not a habit worth attaining.

Here:—(1) The inference in general is invalid; for a thing may be worth acquiring though it does not secure any of those advantages enumerated. (2) The disjunction is incomplete; for there are other goods which virtue insures, though it may not

[1] Hamilton's *Lectures*, Vol. III. p. 350.

insure those here opposed. (3) The subsumption is also vicious; for virtue has frequently obtained for its possessors the very advantages here denied."—*Hamilton*, Vol. III. pp. 352—3.

C.—A Note on Mixed Syllogisms regarded by some Logicians as Immediate Inferences.

Hamilton in his later writings regards Mixed Syllogisms (the Hypothetical and Disjunctive Syllogisms, &c., of Logicians) as Immediate Inferences.

He says:—"It has been a matter of dispute among logicians, whether the class which I call *explicative* (*viz.* the Hypothetical and Disjunctive Syllogisms) be of Mediate or Immediate Inference. The immense majority hold them to be mediate; a small minority, of which I recollect only the names of Kant [Fisher, Weiss, Bouterwek, Herbart], hold them to be immediate. The dispute is solved by a distinction. Categorical inference is mediate, the medium of conclusion being a *term;* the Hypothetical and Disjunctive Syllogisms are mediate, the medium of conclusion being a *proposition*,—that which I call the *Explication.* So far they both agree in being mediate, but they differ in four points. The first, that the medium of the Comparative syllogism is a term; of the Explicative, a proposition. The second, that the medium of the Comparative is one; of the Explicative, more than one. The third, that in the Comparative the medium is always the same; in the Explicative, it varies according to the various conclusion. The fourth, that in the Comparative the medium never enters the conclusion; whereas, in the Explicative, the same proposition is reciprocally medium or conclusion[1]."

Again, (1) "They (Hypothetical and Disjunctive Syllogisms) are not composite by contrast to the regular syllogism, but more simple; (2) if inferences at all, they are *immediate* and not

[1] *Lectures*, Vol. IV. p. 378.

mediate; (3) but they are not argumentations but preparations (explications) for argumentation[1]."

Hamilton gives the following examples among others:—

"A.—CONJUNCTIVE HYPOTHETICALS.

1.	If A be D, A is C;	∴	A, being D, is C. A, not being C, is not D.
2.	If B be A, B is not non-A;	∴	B, being A, is not non-A. B, being non-A, is not A.
3.	If B be not A, B is non-A;	∴	B, not being A, is non-A. B, being non-A, is not A.
4.	If E be not D, E is not C;	∴	E, not being D, is not C. E, being C, is D.

B.—DISJUNCTIVE HYPOTHETICALS.

If B be either A or non-A;	∴	B, being A, is not non-A. B, being non-A, is not A.

'*If*' means *suppose that, in case that, on the supposition, hypothesis, under the condition, under the thought that, it being supposed possible;* ∴ &c., means *then, therefore, in that case,* &c. &c. *actually either*[2]."

Following Hamilton and others, Professor Bain treats of Hypothetical and Disjunctive Syllogisms under the head of Immediate Inferences, and exhibits them as follows:—

"In the Conditional Proposition—If A is B, C is D, the equivalent is, A being assumed to be B, it follows that C is D. There is no inference in this case. Accepting 'A is B,' we accept 'C is D'; this is another expression for the same fact." . . . "A second form of so-called conditional inference is, that the denial of the consequent is the denial of the antecedent; 'C is not D, therefore A is not B.' If the weather is fine, we go to the country; 'we are not going to the country, therefore, the weather is not fine.' This is still mere formal equivalence. It is implied in what has already been stated. It is not a distinct fact but the same fact, in obverse[3]."

[1] *Lectures*, Vol. IV. p. 383. [2] *Ibid.* pp. 390—91.
[3] Bain's *Deduction*, 2nd ed., p. 117.

"The Disjunctive Proposition may appear in the following forms:—

"I. A is either B or C.
II. Either B or C exists.
III. Either A is B or C is D.

"'He is either a fool or a rogue,' means 'If not a fool, he is a rogue, and if not a rogue, he is a fool.' Otherwise, 'Not being a fool, he is a rogue,' and 'not being a rogue, he is a fool.' These are all equivalent forms; and the supposed reasoning consists merely in electing one alternative, according to the facts of the case. The datum being, 'he is not a fool,' we use the alternative 'he is a rogue,' and so on[1]."

"The Dilemma combines a conditional and a disjunctive proposition. If the *antecedent* of a conditional is made disjunctive, there emerges what Whately calls a simple *Constructive Dilemma.* If either A or B is, C is; now, either A or B is; therefore C is." "The *consequent* being made disjunctive, gives the more usual type:—If A is, either B or C is. If the barometer falls, there will be either wind or rain. Various suppositions may be made, bringing out the possible alternatives. Thus:—

(1) A is; then, B or C is.
(2) C is not; then, if A is, B is.
(3) C is; then, if A is, B is not.
(4) B is; then, if A is, C is not.
(5) B is not; then, if A is, C is.
(6) B is not and C is not; then, A is not.

"This last (6) is the *true dilemma* which is *Destructive.*"

"Another form of simple dilemma is:—If B is, A is; and if C is, A is. Now, either B or C is. Whence A is[2]."

That Mixed Syllogisms are mediate inferences and not immediate, will be evident from the following considerations:—

I. In a mixed syllogism there are *three propositions*,—namely, the two premisses and the conclusion,—as in a pure syllogism. The conclusion does not follow from one premiss alone but from

[1] Bain's *Deduction*, 2nd ed., p. 119. [2] *Ibid.* p. 121.

the two taken together. In a hypothetical-categorical syllogism, for example, the major premiss is a hypothetical proposition, the minor premiss a categorical one, and the conclusion also a categorical: "If A is, B is; A is; therefore B is;" here the major premiss expresses the *dependence* of the existence of B on the existence of A, and is not a combination of two propositions as erroneously maintained by some logicians. The minor premiss 'A is' is a categorical proposition, affirming that A exists. It is not the same as the antecedent of the major premiss, which expresses the mere idea, thought, or simple apprehension of the existence of A. It is a proposition with a subject and a predicate, while the antecedent of the major premiss is merely a many-worded term. The two can not be regarded as identical, unless a term and a proposition are identical. The conclusion 'B is' is likewise not the same as the consequent of the major premiss. It is a categorical proposition affirming that B exists, while the consequent is a many-worded term, expressing the mere idea, thought, or simple apprehension of the existence of B.

The major premiss does not affirm that A exists or that B exists. Its antecedent and consequent are not two categorical propositions, but two many-worded terms. It expresses only the relation of dependence of the consequent on the antecedent, and says nothing as to the real existence of either. It lays down the general rule that wherever A is, B is,—that the existence of B accompanies every case of the existence of A. The minor premiss 'A is' asserts that *this* is a case of the existence of A. Whence it is inferred that there is a case of the existence of B, accompanying *this* case of the existence of A, or, in other words, that 'B is' (conclusion).

The minor premiss may be taken as a hypothetical proposition, with 'this case' understood for its antecedent; thus, "if this case is, A is." From this and the original hypothetical major premiss follows the conclusion, that "if this case is, B is," or, in other words, that 'B is' (conclusion), taken as a hypothetical proposition with 'this case' understood for its antecedent.

In the *destructive* form "If A is, B is; B is not; therefore A is not," the major premiss is hypothetical, and the minor premiss and the conclusion are categorical propositions as in the *constructive form*. The differences between the two forms are (1) that the minor premiss and the conclusion are affirmative in the constructive form, and negative in the destructive, and (2) that the minor premiss of the one and the conclusion of the other have the same subject and predicate, but *differ in quality*. Thus (1) the two affirmative propositions 'A is' and 'B is' are the minor premiss and the conclusion, respectively, in the constructive form, and the two negative propositions 'B is not' and 'A is not' are the minor premiss and the conclusion, respectively, in the destructive form. (2) 'A is' is the minor premiss in the constructive form, and 'A is not' is the conclusion in the destructive form; in the former 'B is' is the conclusion, and in the latter 'B is not' is the minor premiss. The conclusion of the one has the same subject and predicate as the minor premiss of the other. From this fact has probably arisen the mistaken notion that in these syllogisms 'the minor premiss and the conclusion indifferently change places[1].' Hamilton says: "The fourth, that in the Comparative the medium never enters the conclusion; whereas in the Explicative (*i.e.* hypothetical syllogisms, &c.) the same proposition is reciprocally medium or conclusion" Now, the proposition is not the same. Its subject and predicate only are the same, but its quality is different. The minor premiss of the one, and the conclusion of the other, can not be regarded as the same, unless an affirmative and a negative proposition, having the same subject and predicate, are the same,—unless A and E, A and O, E and I, I and O, are identical. With equal justice might the conclusion in one, and the minor premiss in the other, of the two forms, namely, *affir-*

[1] This point is differently interpreted by Professor Robertson (*Mind* for 1877, p. 264) and Mr Keynes (*Formal Logic*, p. 234). They consider it to be a blunder, from which, I think, Hamilton is free, as is evident from the examples given by him and quoted in this book on page 289.

mative and *negative*, of the following categorical syllogisms, be regarded as identical:—

1. *Affirmative* Categoricals:—

(1) All men are mortal,
All kings are men,
∴ All kings are mortal.

(2) All men are mortal,
Some kings are men,
∴ Some kings are mortal.

2. *Negative* Categoricals:—

(1) All men are mortal,
All kings are not mortal,
∴ All kings are not men.

(2) All men are mortal,
No kings are mortal,
∴ No kings are men.

1. Corresponding *Constructive* Hypothetical-categoricals:—

(1) If all kings are men, all kings are mortal;
All kings are men;
∴ All kings are mortal.

(2) If some kings are men, some kings are mortal;
Some kings are men;
∴ Some kings are mortal.

2. Corresponding *Destructive* Hypothetical-categoricals:—

(1) If all kings are men, all kings are mortal;
All kings are not mortal,
∴ All kings are not men.

(2) If some kings are men, some kings are mortal;
No kings are mortal,
∴ No kings are men.

The minor premiss in one and the conclusion in the other of the *affirmative* and *negative* categoricals have the same subject and predicate, and stand to each other in the same relation in which the minor premiss in one and the conclusion in the other of the *constructive* and *destructive* hypothetical-categoricals stand to each other. But who would maintain that in those categorical syllogisms, "the minor and the conclusion indifferently change places," or that "the same proposition is reciprocally medium or conclusion"?

II. In a mixed syllogism there are *three terms* as in a pure syllogism. In the example taken above, the consequent as a many-worded term, is the major term, the antecedent as a many-worded term, is the middle term, and 'this case' or 'the case in

question' *understood*, is the minor term. This will be evident, if the mixed syllogism is reduced to the *pure* form:—

(i) Categorical:

Every case of the existence of A is a case of the existence of B; the case in question (or this case) is a case of the existence of A; therefore the case in question (or this case) is a case of the existence of B.

Here the three terms are—(1) case of the existence of B (major term), (2) case of the existence of A (middle term), and (3) the case in question or this case (minor term). (2) is the middle term to which (1) and (3), the two extremes, are related;—that is, a relation between (1) and (3) is established from a relation of each of them to a third (2) or middle term, as in the case of a categorical syllogism.

(ii) Hypothetical:

If A is, B is; if this case is, A is: therefore if this case is, B is.

This is a pure hypothetical syllogism in *Barbara*. Here the middle term is the antecedent in the major premiss, and consequent in the minor, as it should be in that mood.

From this it is evident, that the objection that a mixed syllogism has no middle term, and consists of two terms only, is entirely unfounded. It has arisen from a misunderstanding of the true nature of the hypothetical major premiss, which has been erroneously regarded as consisting of two propositions instead of two many-worded terms. It is also evident that the middle term is not, as Hamilton says, a proposition, but a many-worded term.

III. If A is B, C is D;
∴ A being B, C is D.

This is the form in which a mixed syllogism regarded as an immediate inference is stated; and it is argued that the conclusion follows immediately from the premiss, and that no minor premiss is necessary. Now, it can be shown that a categorical syllogism may likewise be stated in the above form;

and should it, therefore, be regarded as an immediate inference?

All men are mortal,
All kings, being men, are mortal.

Here also the conclusion follows from the premiss. But it is evident that the conclusion is but a short or abridged statement of two propositions, namely, the minor premiss, 'all kings are men,' and the conclusion, 'all kings are mortal.' Some logicians indeed actually maintained that even in the categorical syllogism, the minor premiss is unnecessary, that the conclusion follows from the major premiss. Thus they would regard categorical syllogisms as consisting of two propositions only, and consequently as immediate and not as mediate inferences. But we have seen (pp. 257—8) that the conclusion does not follow from the major premiss alone, nor from the minor alone, but from the major and the minor taken jointly. And this is true of mixed syllogisms as well as of categoricals. The conclusion 'A being B, C is D,' is merely a short or abridged statement of two propositions, namely, the minor premiss 'A is B,' and the conclusion 'C is D.'

Here may be noticed an objection raised by Professor Bain. He sees no *real* inference in mixed syllogisms. By real inference he means a proposition that is not contained in, or implied by, the premiss or premisses. This objection is founded on a misunderstanding of the true nature of deductive inference. It is equally applicable to categorical syllogisms. In these also the conclusion is not a *real* inference, but a proposition which is contained in, or implied by, the two premisses. Without disputing about words, it may be said that the inference is *mediate* and *real* in mixed syllogisms, if it is mediate and real in categoricals.

D.—A Note on the Reduction of Inductive Reasoning to the Syllogistic Form.

The fundamental principles of Inductive Reasoning (whatever be their origin and nature) are the two Laws of Causation and Uniformity of Nature. The first law includes the two propositions—(1) every phenomenon has a cause; and (2) the cause of a phenomenon is the invariable, or, as Mill says, the unconditionally invariable antecedent of the phenomenon. The second law means that (3) the same cause or antecedent will, under the same circumstances, produce the same effect. All inductive reasonings are conducted either directly in accordance with one or other of these laws or with laws that follow from them. For example, from the second proposition of the first law follow such laws as the following given by Professor Bain[1]: (4) 'whatever antecedent *can be left out*, without prejudice to the effect, can be no part of the cause;' (5) 'when an antecedent *can not be left out* without the consequent disappearing, such antecedent must be the cause or a part of the cause;' (6) 'an antecedent and a consequent rising and falling together in *numerical concomitance* are to be held as cause and effect,' and also the following: (7) 'if two or more instances of a phenomenon under investigation have only one circumstance in common, that circumstance is the cause (or effect) of the phenomenon;' (8) 'if an instance where a phenomenon occurs, and an instance where it does not occur, have every circumstance in common except one, that one occurring only in the first; the circumstance present in the first and absent in the second, is the cause, or a part of the cause, of the given phenomenon'[2].

[1] Bain's *Induction*, 2nd ed., pp. 47, 48, 57.

[2] That the propositions marked (4), (5), (6), (7), and (8) follow from the proposition marked (2) can be shown as follows:—

(4) is the converse of the obverse of (2). Obvert (2), and then convert the obverse;—the cause of a phenomenon is not the variable antecedent of the phenomenon—[E, obverse of (2)]. (4) That which

Examples of Inductive Reasoning:—

(1) The antecedents		A B C	produce the consequents		a b c
"	"	A B D	"	"	a b d
"	"	A D E	"	"	a d e
"	"	A E F	"	"	a e f

∴ The antecedent *A* is the cause of the phenomenon *a* according to the principle—a derivative one—marked (7) above, and called the Canon of the Method of Agreement. This inductive reasoning may be easily reduced to a syllogism which has for its major premiss the canon, and for its minor the data of the reasoning, that is, the instances of the phenomenon. The syllogism is a hypothetical-categorical one, and is as follows:—

If two or more instances of a phenomenon under investigation have only one circumstance in common, that circumstance is the cause of the phenomenon (major premiss).

The four instances given of the phenomenon *a* under investigation have only one circumstance, namely, *A*, in common (minor premiss).

is the variable antecedent of a phenomenon, or, in other words, which 'can be left out without prejudice to the effect,' is not the cause of the phenomenon (E, converse of the obverse).

(5) is the converse of (2), which, being a *definition*, may be converted *simply*. (5) That which is the invariable antecedent of a phenomenon, or, in other words, which 'can not be left out without the consequent disappearing,' is the cause of the phenomenon [A, converse of (2)].

(6) is a mathematical inference from (2). The cause and the effect increase or decrease together. A = B ∴ 2A = 2 B, or nA = nB.

(7) follows from (4) and (5) taken together. By (4) the circumstances which are not common to all the instances of the phenomenon, that is, which 'can be left out without prejudice to the effect,' can be no part of the cause. By (5) the circumstance which is common to all the instances, that is, which 'cannot be left out without the consequent disappearing,' is the cause or a part of the cause of the phenomenon.

(8) follows likewise from (4) and (5) taken together.

∴ That circumstance *A* is the cause of the phenomenon *a* (the conclusion).

Or, the syllogism may be stated in the form of a categorical as follows :—

The invariable antecedent of a phenomenon is the cause of the phenomenon (major premiss).

A is the invariable antecedent of the phenomenon *a* (minor premiss).

∴ *A* is the cause of the phenomenon *a* (the conclusion).

(2) The antecedents A B C produce a b c
 „ „ B C „ b c,

∴ The antecedent *A* is the cause or a part of the cause of the phenomenon *a* according to the principle—also a derivative one—marked (8) above, and called the Canon of the Method of Difference. This inductive reasoning may be likewise reduced to the syllogistic form as follows :—

If an instance where a phenomenon occurs, and an instance where it does not occur, have every circumstance in common except one, that one occurring only in the first; the circumstance present in the first and absent in the second is the cause, or a part of the cause, of the given phenomenon (major premiss).

An instance A B C......a b c, where the phenomenon *a* occurs, and an instance B C......b c, where it does not occur, have every circumstance in common except one, namely, *A*, that one occurring only in the first (minor premiss).

Therefore, the circumstance *A* present in the first and absent in the second is the cause, or a part of the cause, of the given phenomenon *a* (conclusion).

Or, as follows :—

When an antecedent can not be left out without the consequent disappearing, such antecedent must be the cause, or a part of the cause, of the consequent (major premiss).

The antecedent *A* can not be left out without the consequent *a* disappearing (minor premiss).

Therefore the antecedent *A* must be the cause, or a part of the cause, of the consequent *a*.

Similarly, other inductive reasonings may be reduced to the syllogistic form.

Let us take as a concrete example the first one we have given in the chapter on the Different Kinds of Reasoning (page 123):—

Air expands by heat,
Water expands by heat,
Mercury expands by heat,
Copper expands by heat,
&c. &c.

∴ All material bodies expand by heat[1].

Here the antecedent circumstances are the material bodies *plus* heat, and the consequents or effects are the same bodies *plus* the phenomenon of expansion. All the antecedents agree in the circumstance of being heated material bodies; and, therefore, according to the Canon of the Method of Agreement, this circumstance is the cause of the phenomenon of expansion, that is, in the given instances, heat being the invariable antecedent of expansion is the cause of this phenomenon. More accurately, the different steps of the argument may be stated as follows:— (1) Air and other bodies expand by heat, the expansion of these bodies is a phenomenon; therefore it has a cause, according to the principle 'every phenomenon has a cause;' (2) the invariable antecedent of this phenomenon is the application of heat, as shown by the given instances; therefore, according to the principle, namely, 'the invariable antecedent of a phenomenon is the cause of the phenomenon,' the application of heat to material bodies is the cause of the expansion in the given instances; and (3) according to the principle, namely, 'the same antecedent or cause will, under the same circumstances, produce the same effect,' it may be inferred that the application of heat to other

[1] This proposition is not universally true. See an exception on page 75. But that does not affect the line of reasoning adopted here.

material bodies, as well as to the same in future, will produce expansion; or, in other words, all material bodies expand by heat. The different steps may be thus stated syllogistically:—

(1) Every phenomenon has a cause, the expansion of air and other bodies by heat is a phenomenon; therefore it has a cause.

(2) The invariable antecedent of a phenomenon is the cause of the phenomenon, the application of heat is the invariable antecedent of the phenomenon of expansion in the given instances; therefore the application of heat is the cause of the phenomenon of expansion in the given instances.

(3) The same antecedent or cause will, under the same circumstances, produce the same effect or consequent,—that is, if a certain antecedent produces, under certain circumstances, a certain consequent, then it will, under the same circumstances, produce the same consequent; the antecedent, namely, the application of heat to material bodies, under the circumstances of there being no counteracting agencies, produces the consequent, namely, the expansion of those bodies; therefore the same antecedent, namely, the application of heat to material bodies, under the same circumstances of there being no counteracting agencies, will produce the same consequent, namely, the expansion of those material bodies.

Thus all inductive reasonings, like mathematical (see p. 123), may be reduced to the syllogistic form: usually their conformity to an axiom, principle, law, canon, or rule recognized as true is regarded as a sufficient proof of their validity, even as constituting their validity itself; but in all cases where they are valid, they are capable of being reduced to the syllogistic form. In Physics, for example, conformity to the principles of causation and of uniformity of nature, or to the canons and rules derived from them, is regarded as constituting the validity of the reasonings; but we have seen that, taking the principles or the canons as major premisses, and the data as minor, we can, in all cases, construct syllogisms which have the same conclusions as the reasonings themselves; and the best test of the validity of the reasonings is the possibility of their reduction to the syllogistic

form: any weakness in the argument is sure to come to light by this process.

To see clearly what premisses have been assumed, or, on what data—both principles and facts—the conclusion ultimately rests, it is necessary to reduce a reasoning or a train of reasoning to the syllogistic form. In this form every step of the argument will be clearly exhibited and every proposition required to prove the conclusion laid bare, and should there be any error in the *process* of reasoning, it will be brought to light by the axioms, canons, or rules of Deductive or Syllogistic Logic. Of course, if there be any falsity or fallacy in the ultimate data—if any universal principle or any particular fact has been unwarrantedly assumed—it can not be detected by those axioms, canons or rules; nor can it be detected by the canons and rules of any Logic, as understood by British Logicians. For the *particular fact*, the ultimate appeal must be made to observation, external or internal; and for the *universal principle* the appeal is made (1) to the Experience of the Individual, that is, to Repeated Experience and Generalisation (the Empirical or Experiential Theory); or (2) to Intuition, that is, to Immediate Knowledge by the Reason (the Intuitional Theory); or (3) to the Forms and Categories of the Mind (the A-priori or Kantian Theory); or (4) to the Experience of the Race, that is, to Inherited Tendencies and Experience (the Evolutional Theory). The first question can be decided only by the special science to which the fact belongs; and the second question by the science which treats of the origin and nature of universal principles, and which has been variously called Metaphysics, the Science of First Principles, the Science of the more General Laws, &c.

E.—The Nature and Province of Objective Logic.

The *name* 'Objective Logic,' and the *thing* signified by it, are comparatively new. I intend, therefore, to give here extracts from the writings of Logicians with a view to indicate the nature and province of the *thing* as conceived by them.

§ 1. *Hamilton's View.*

"The doctrine...which expounds the laws by which our scientific procedure should be governed, in so far as these lie in the forms of thought, or in the conditions of the mind itself, which is the subject in which knowledge inheres,—this Science may be called *Formal*, or *Subjective*, or *Abstract*, or *Pure* Logic. The Science, again, which expounds the laws by which our scientific procedure should be governed, in so far as these lie in the contents, materials, or objects, about which Knowledge is conversant,—this Science may be called *Material*, or *Objective*, or *Concrete*, or *Applied* Logic[1]."

§ 2. *Mill's View.*

In Mill's writings the *name* 'objective Logic' rarely, if ever, occurs; but the *thing* is to be found in abundance. He defines and treats of the *thing* in his *Examination of Hamilton's Philosophy* and also in his *System of Logic*, and expounds and criticises logical doctrines from that point of view. There is, however, a difference between the *thing* as conceived in the *Examination*, and the *thing* as treated of in the *Logic*. In the former he speaks of concepts, judgments, and reasonings, and requires that they should be right or true, that is, that they should *agree* with fact or reality. In the latter he treats of phenomena or facts themselves: names, for instance, stand for things; propositions for relations of things; and arguments are about the relations of those relations. In the *Logic* he gives up

1 Hamilton's *Lectures*, Vol. IV. p. 231.

concepts and judgments, and condemns the theories of predication, which are founded upon ideas of things, and not upon things or phenomena themselves. The *Logic*, therefore, treats of things and their relations; and it is from this point of view that he finds the Syllogism guilty of the *petitio principii*, and Immediate Inference as no inference at all.

Mill's conception of Logic has thus two phases:—

(1) In the first phase Logic is conceived to treat of concepts, judgments, and reasonings as *agreeing* with things.

(2) In the second phase, Logic is conceived to treat of things or phenomena themselves, and of their relations and correlations.

Among English Logicians Mill, in fact, seems to occupy an intermediate position between such Subjective Logicians as Hamilton and Mansel, and such Objective Logicians as Spencer and Lewes[1].

§ 3. *Spencer's View.*

"A distinction exists which, in consequence of its highly abstract nature, is not easily perceived, between the science of Logic and an account of the process of Reasoning......The distinction is, in brief, this, that Logic formulates the most general laws of correlation among existences considered as objective; while an account of the process of Reasoning, formulates the most general laws of correlation among the ideas corresponding to those existences. The one contemplates in its propositions, certain connexions predicated, which are necessarily involved with certain other connexions given; regarding all these connexions as existing in the *nonego*—not, it may be, under the form in which we

[1] On the difference between Formal Logic (Hamilton's view) and Material Logic (the first phase of Mill's view of Logic), see Venn, *Logic of Chance*, 2nd ed., chapter x., "Discussion of some of the Principal Views as to the Nature and Province of Logic, Material and Conceptualist." On the difference between the two phases, briefly indicated above, of Mill's conception of Logic, compare Ueberweg's distinction of Logic and Metaphysics. See *Logic*, §§ 1, 2, 3, 8.

know them, but in some form. The other contemplates the process in the *ego* by which these necessities of connexion come to be recognised.

"Why this distinction has eluded observation, it is not difficult to see. Logic on the one hand, and the theory of Reasoning on the other, deal with relations from which all concrete terms are, as far as possible, expelled. They are severally obliged to use some terms (which, however, are by preference symbolic, so that they may express indifferently any kind of existence, attribute, action, or even relation); otherwise the relations dealt with can not be expressed, or distinguished from one another. But they intentionally ignore the natures of the terms, and occupy themselves with the most general dependencies of these most abstract relations. The result is that, in the absence of terms definitely specified as belonging either to the outer world or to the inner world, the two sets of relations, belonging the one to the outer world and the other to the inner world, become indistinguishable. Hence there arises this confusion between Logic, which is as much a division of the science of objective existence as Mathematics, and the theory of Reasoning, which is a division of subjective Science.

"To show that the affirmations of Logic refer to the connexions among things considered as existing apart from our consciousness, and not to the correlative connexions among our correlative states of consciousness, we need but to take the case of logical propositions as numerically quantified, in the system of Prof. de Morgan. I quote Mr Mill's condensed statement of the doctrine; for Prof. de Morgan's own statements are so encumbered with details and symbols, that I can not find in his work one that is at once brief and adequate.

"'From the premises most B's are C's, most B's are A's, it may be concluded with certainty that some A's are C's, since two portions of the class B, each of them comprising more than half, must necessarily in part consist of the same individuals. Following out this line of thought, it is equally evident that if we knew exactly what proportion the 'most' in each of the premises

bear to the entire class B, we could increase in corresponding degree the definiteness of the conclusion. Thus if 60 per cent. of B are included in C, and 70 per cent. in A, 30 per cent. at least must be common to both; in other words, the number of A's which are B's, and of C's which are A's must be at least equal to 30 per cent. of the Class B.'[1]"

"......But the clearest proof that relations among objective existences form the subject-matter of Logic, is yielded by the mechanical performance of logical inference. Prof. Jevons has devised a machine of such kind that, its keys being pressed down in proper order in conformity with the premisses of the given logical proposition, the conclusion is presented by the combinations which the machine displays. Here it is undeniable that the relation disclosed is an objective one; and it is equally undeniable that the thing ascertained is, that this objective relation was necessarily involved in those other objective relations which constitute the premisses. We have nothing to do with thought at all. We have to do with inter-dependencies among outer things or agencies. The machine having been set to represent objects and attributes in certain relations, evolves certain necessarily-accompanying relations, such as would otherwise be ascertained by actual examination of the objects and attributes[2]."

"The propositions of Logic, then, primarily express necessary dependencies of things, and not necessary dependencies of thoughts; and in so far as they express necessary dependencies of thoughts, they do this secondarily—they do it in so far as the dependencies of thoughts have been moulded into correspondence with the dependencies of things. I say advisedly, '*in so far as*'; for there are certain absolute unlikeness of nature between the outer dependencies and the inner dependencies which for ever forbid anything more than a symbolic correspondence, as we shall hereafter see more clearly. The greater part

[1] *Principles of Psychology*, 2nd ed. Vol. II. § 302, pp. 87—88.

[2] *Principles of Psychology*, Vol. II. § 302, p. 90.

of the necessary objective correlations are *statical*, while all the necessary subjective correlations are dynamical; and only in so far as dynamical correlations may be so arranged as to symbolize statical correlations, can the necessary dependencies of Reason be made to parallel the necessary dependencies of Logic[1]".

"......See, then, the inevitable implication. No one questions the fact that while I was using these marbles to exemplify arithmetical truths and geometrical truths, I was contemplating, and was teaching, necessary objective correlations. Can it be that when I used these same marbles to exemplify necessities of correlation among groups and sub-groups, distinguished by certain marks, I passed from the region of objective necessities to the region of subjective necessities? No one will, I think, have the hardihood to assert as much. There is no choice but to leave these most general laws of correlation which Logic formulates, outside along with the laws of numerical correlation and geometrical correlation; or else, bringing them into the mind as laws of thought, to bring with them these mathematical laws as laws of thought in the same sense, and, by other steps equally unavoidable, to merge all objective facts in subjective facts: thus abolishing the distinction between subject and object[2]".

Note. Mr Carveth Read adopts Spencer's view of Logic, with these two qualifications, *first*, that Logic "may very well consider the correlation of ideas among themselves," and *second*, that Logic "deals only with laws of phenomena." See *Mind*, Vol. II. "On some Principles of Logic," p. 336. For a Critical Notice of Mr Read's "*Theory of Logic: an Essay*," by Dr Venn, see *Mind*, Vol. III. p. 539. See also a note on "'Matter-of-fact' Logic," by Mr J. N. Keynes, in *Mind*, Vol. IV. p. 120. For a criticism of Spencer's view of Logic, by Dr Venn, see *Mind*, Vol. IV., "The Difficulties of Material Logic," p. 35. Dr Venn suggests a view of Logic which seems to correspond to Ueberweg's view and to the first phase of Mill's conception of Logic (see Mill's View). Instead of regarding Logic as a purely objective

[1] *Principles of Psychology*, Vol. II. § 302, pp. 90—91.
[2] *Principles of Psychology*, Vol. II. § 302, pp. 92, 93.

science," says Dr Venn, "we might with more propriety term it a science which gives the rules for converting the subjective into the objective" (*Mind*, Vol. IV. p. 46). Compare Ueberweg's definition, namely, "Logic is the science of the regulative laws of human knowledge" (*Logic* § 1), and Mill's view of Logic as "the science of the conditions on which right concepts, judgments, and reasonings depend" (*Examination of Hamilton's Philosophy*, 4th ed. p. 464).

§ 4. *Lewes's View.*

"Let us pause for a moment to consider the very different meanings assigned to the word Logic. It commonly stands for:

(1) the art of reasoning;

(2) the theory of reasoning;

(3) Reasoning itself;

(4) the laws of mental operation, irrespective of the symbols operated on (Formal Logic);

(5) the rules of Proof.

"The first of these I hold to be absurd. There is no more an art of Reasoning than there is an art of Breathing, or Digesting. But so little is this understood that even thoughtful writers will be found declaring that we must learn how to reason, as we learn how to fence or to swim. In consequence of this misconception, certain studies, notably Mathematics, are popularly believed 'to strengthen the Faculty,' to develop the logical powers, to 'invigorate the judgment.' The psychological notions which lie at the basis of such declarations are sadly defective.

"The second and third meanings of the word are objectionable because restricting Logic to the process of Ratiocination when the ratios are abstract. This restriction is got rid of in the fourth and fifth meanings, which may be accepted as comprehensive. The fourth designates the universal Logic, it includes all Laws of Grouping (λέγειν means to bind together, to group), and is therefore applicable to Feeling and Thought (in the subjective world), and to Cause (in the objective world).

"The fifth has the technical and restricted meaning of a *Codification of the rules of Proof.* In this last sense only can Logic be a separate Discipline. It may be likened to the science of Grammar apart from Language. Thus the speech of men of various nations embodies and exhibits certain general rules; or tendencies, according to which words are grouped. These tendencies grammarians detach and treat separately as Laws of Speech, Rules of Grammar. Logicians may in like manner detach certain general procedures of the investigating intellect, and treat them apart as the Rules of Rational Inquiry.

"Having fixed on the meaning Logic may bear when employed for a Special Discipline, namely, the codification of the rules of Proof, we may complete it by assigning to Metaphysics the parallel position of a codification of the laws of Cause. It will thus occupy very much the place assigned to it by Hegel, namely, that of Objective Logic. The Object and the Subject would have one general Logic, separately viewed as the Logic of Intelligence, and the Logic of the Cosmos. In the Cosmos, viewed objectively, things influence each other and events succeed each other according to invariant tendencies, or laws. When these phenomena are reproduced in consciousness they are also reproduced according to invariant tendencies; and thus it is that a law of Cause becomes a rule of Proof. Logic in its widest sense is Grouping. The laws of Grouping are the general tendencies of Things and the general tendencies of Thought. The common separation of Thought from the things thought of, is an artifice; but it is one so deeply inwoven with our philosophy and practice, that the mind untutored in such researches, is astonished and distressed at the statement of the identity between Thing and Thought, Object and Subject. With what qualifications this statement has to be received we shall hereafter discuss. Here I am only concerned to define the position of Metaphysics as Objective Logic—the codification of the most abstract laws of Cause. The Subjective Logic takes no account of the special instruments and processes by which each science reaches Proof, it is occupied solely with the codification of the processes. In

like manner the Objective Logic disregards special details in the processes of Causation, solely occupied with codifying the most abstract results. Subjective Logic rejects whatever lies beyond the range of verification, and thus demarcates Reality from Possibility, Fact from Fiction. Objective Logic rejects whatever lies beyond that world of sensibles and extra-sensibles which can come within the range of Experience; and thus demarcates Metaphysics from Metempirics.

"This distinction between the two aspects of Logic represents the distinction between Knowing and Being; and the identity underlying this diversity is also represented. In one we find the laws of Investigation; the abstract conditions to which all knowledge is subject. In the other we find the laws of the Investigated, the abstract conditions to which the knowledge is subject. Only on the assumption of the invariability of relations objective and subjective is Philosophy possible. In the most abstract of the sciences, that of Number, this identity is manifest. No arithmetical operation would be valid were there not this accord between the internal and the external; and the assumption of such an accord runs throughout Science. Indeed the axioms of Logic and the axioms of Science are the concave and convex aspects of the same curve[1]."

In a footnote to the above, Lewes remarks:—"Since this view was written Mr Spencer has propounded a new view of Logic. Starting from the proposition that the Syllogism refers to the dependencies of Things and not of Thoughts, he comes to the conclusion that Logic must be carried over entirely to the Objective world. He therefore places it beside Mathematics—as it is placed in Comte's latest scheme. He holds that 'it formulates the most general laws of correlation among existences considered as objective.' Referring the reader to Mr Spencer's exposition (*Psychology*, II. §§ 302 *et seq.*), I will merely here add that my chief divergence from it arises from my inability to

[1] Lewes's *Problems of Life and Mind*, 3rd ed. Vol. I. pp. 72—75.

accept his conception of there being only a symbolic correspondence between the inner and outer worlds. I hope to make it clear that the correspondence is real[1]."

§ 5. *Summary.*

According to Hamilton, Objective Logic is the science of the forms of the objects known, and Subjective Logic the science of the forms of the Knowing Subject. According to Spencer, Logic is the science of "the most general laws of correlation among existences considered as objective," and the Theory of Reasoning the science of "the most general laws of correlation among the ideas corresponding to these existences." Spencer's Logic and Theory of Reasoning seem to correspond to Hamilton's Objective Logic and Subjective Logic, respectively. According to Spencer, Logic, like Mathematics, is an objective science, and treats of the most general laws of objects existing in the outer world. It is as little dependent upon mental processes as Mathematics. Its processes and laws are determined by the processes and laws of objects and not of thoughts.

Lewes regards Objective Logic as identical with Metaphysics. "The Object and the Subject would have one general Logic, separately viewed as the Logic of Intelligence and the Logic of the Cosmos." This general Logic is Objective Logic applicable alike to the Subject and to the Object, to both thoughts and things. Subjective Logic is concerned, according to him, with the codification of the rules of Proof, of the processes of Knowing, and Objective Logic with the codification of the most abstract laws of Cause, of the processes of Being. This distinction between Subjective and Objective Logic seems to correspond to Hamilton's and Spencer's distinction of these two Logics.

According to Lewes, Thought and Things, Knowledge and Being are, like the concave and convex aspects of the same curve, the subjective and objective aspects of the same existence; and the Logic of the one really corresponds to, or is identical

[1] *Problems of Life and Mind*, 3rd ed. Vol. I. p. 75.

with, the Logic of the other. While, according to Spencer, the Subject and the Object, the Ego and the Non-ego are two separate realities; and the Logic of the one has only a certain symbolic correspondence or parallelism to the Logic of the other.

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